CLASSROOM PROJECTS with ENVIRONMENTAL APPLICATIONS

Faculty Classroom Guide

A Versatile Technology-Intensive Earth Math

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Welcome to the Earth Math Projects

The Earth Math project provides you with a new way to integrate environmental science data and real life problems into your classroom. Whether you are familiar with the authors of the Earth Algebra texts or new to this project, this faculty classroom guide is designed to help you in several ways. You will want to use the guide because you…

➢ want to become familiar with the Earth Math web site and tools.
➢ are preparing for instruction with the Earth Math modules.
➢ need to take note of the ways you might want to use the Earth Math modules as you teach your class.
➢ want to learn to use the Earth Math demonstration materials, modules, and applets.
➢ wish to use the guide as a reference throughout the term and beyond.

We would suggest you print this guide to have a hard copy reference to the site materials and uses. It will provide you a map to the materials, descriptions, and directions.

Notes to the Instructor

The content of this handbook reflects the third version of the Earth Math modules. This guidebook will be updated to reflect the changes made to the modules and applets as the project evolves. There may be minor differences in the directions given in this guidebook compared to the web site as the project is continued. Faculty designed this guidebook for faculty and we welcome your suggestions and comments, so that we may improve the training manual. Send all suggestions and comments, as well as ask assistance when you need it, to:

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Please contact Lynn if you have any questions related to the Earth Math training manual.
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The Earth Math Projects are an exciting extension of the previous mathematics textbooks and materials produced with applications to Environmental Science.

Over the past 12 years, the Earth Math Projects have produced unique materials that teach mathematics through the study of environmental issues and problems. The materials are appropriate for use in mathematics courses ranging from beginning algebra through calculus.

Three textbooks have resulted from these projects, *Earth Algebra* (algebra), Addison/Wesley, *Earth Angles* (precalculus), Addison/Wesley, and *Earth Studies* (applied calculus), Kendall/Hunt. The applications are based on real environmental problems that effect students' lives and therefore generate more interest in the use of mathematics as a tool that can be used to analyze real situations. The materials have been tested and formally evaluated and have shown that the use of mathematics to study real problems that are interesting to students significantly improves interest in, understanding and appreciation of the role of mathematics in science and society.

As a spin-off, special class notes based on water availability for a growing community were developed for Diné College for use in the college algebra course; the success rate after introduction of these notes improved from 50% to 70%.

**Current Project Activities**
The National Science Foundation and the U.S. Dept. of Education, FIPSE have funded Schaufele and Zumoff, for a curriculum development project which was an extension of the work described above. The goal is to produce versatile, technology-intensive materials for classroom use and teacher training. Reform-based applications are incorporated into platform-independent software to make them accessible to anyone with a computer. The versatility of these materials will allow an instructor to use them regardless of textbook or underlying curricula.

Environmental applications from the previous projects and new applications are designed for courses from beginning algebra through calculus. The project features an inquiry-based format, web-based interactive materials, seamless interface with state-of-the-art technology, use of real data, interesting applications of mathematical concepts, and flexibility of classroom use.

**Materials**
Each module, or study, is presented on the Web (also available on CD-Rom) and can be used as an in-class or lab project that illustrates real applications of mathematics to students. Students are able to access data provided on the website or download relevant data from the Internet. Software applets are written that can perform mathematical operations reflecting concepts learned in the classroom or be used for demonstration purposes. These tools are available from the Web and packaged on the CD. Review topics of related prerequisite skills are linked to the modules to provide a quick, easy view of content necessary to complete the study.

**Resources**
Resources for the faculty using the Earth Math modules include links to data, other related projects, references, announcements of user activities, as well as contact with authors and others involved in the project consortium.
Chapter 1    Web Site Specifics

Home Page and Site Map
This section of the guidebook gives an overview of information regarding the web site, using the web site buttons, and all its materials.

FAQ   What does the web site include?

Becoming Familiar with the Earth Math Web Site

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Instructors using Earth Math modules will find a description of the Earth Math homepage and the site for easy reference and use. In this first chapter, the instructor materials and course materials will be discussed, as well as the site evaluation and links for resources and use in the classroom. The next chapter will describe the content of the modules and the use of each of the site tools.
Introduction
The Earth Math homepage has several components, the most important of which is the link to the instructor course materials. Instructors entering the site for the first time will notice the easy-to-follow site composition. You will notice the components in the home page below. The Earth Math homepage has several Instructor Materials links to assist the instructor in the use of the Earth Studies. Each of these links is identified in this section, with a brief description of the materials found on the page. The design of the home page allows easy access to the materials, with the home page buttons found on each course page as well.

Standard Menu Buttons: Left Screen List
The menu to the left side of the homepage is always in this place on the Earth Math web pages. This allows easy access to the most commonly needed pages and tools, as well as easy movement from one part of the site to another. These buttons allow access to the Instructor’s Material to include the Faculty Classroom Guide, Tool Chest and Review Topics; Earth Studies modules by level and use; and links to the Navajo Nation and Evaluation summaries. A brief description of the materials for each of the buttons follows.
Instructor’s Material Buttons
A description of each of the materials buttons for the instructor using the Earth Math materials is given in the following text. The descriptions follow the site composition, beginning with the Instructor’s Materials links, down the left side of the homepage. Each of these buttons is a link to the materials for that topic. The guide will explain the materials for each of these links to site materials. Click on the button to access the menu for each of these useful materials.

Guide to Studies
The Earth Math faculty classroom guide is designed to assist in the use of the Earth Math Projects modules. A complete description of the Earth Math materials and applet tools, as well as instructions for use, is contained within this guide. The guide is useful to both the new user of Earth Math Projects and those who are familiar with the materials. The guidebook may be used online, or printed to use as a desk reference. The guide is provided as a Word or PDF file.

Tool Chest
The Earth Math Tool Chest is a link to the applet tools available for classroom instruction. These tools may be used for classroom demonstration or analysis, by both the student and instructor. This link allows easy access to all the tools without having to first go into a module, making them available whether using the Earth Studies or other materials. The Applet tools include Computation Applets for computation of expressions and equations, plotting, and regression; Instructional Applets for plotting graphs of linear, exponential, quadratic, and trigonometric functions as well as other demonstrations; and Experimentation Applets for the student to experiment with ‘best fit’ models of linear, quadratic, exponential, and trigonometric functions.

Applet tools open in a new window, so the student or instructor can access both the module information and the tool. A short description of the specific tools is given in Chapter 2, Using the Applets and a complete description of the web Tool Chest is given in Chapter 3, including directions for using the tools. Tools are available as Computation Applets, Instructional Applets, and Experimentation Applets.

Applet tools are written by Dr. Philippe Laval, Kennesaw State University
Review Topics
The Earth Math Projects include a menu of review topics, designed to assist the student in the review of important prerequisite concepts for successful completion of the Earth Math Projects Modules. Each of the review topics includes the general symbolic forms, conceptual visualization, and examples to clarify understanding. These topics are accessible from the modules but are also included in the standard menu to allow easy access for ‘just in time’ review of concepts for the student.

The Review Topics open in a new window and have some of the applet tools embedded into the topic, to clarify the concepts and mathematical understanding. A short description of the specific review topics is given in Chapter 2, Using Review Topics and a complete description of the review topics is given in Chapter 3, including the general symbolic language of the topic. Review topics for Algebra, Pre-calculus, and Calculus levels are available.

Review Topics are written by Marlene Sims, Kennesaw State University.

Earth Studies
The Earth Math homepage has links to Earth Math materials for each of the levels of study written into the projects. The modules are written to assume some basic skill level for the courses that are ordinarily taught at the listed level. The Introductory Material is included to learn how to use the Earth Math modules and tools, therefore have additional material included for instruction. Objectives for each study are included in the module, with descriptions given in Chapter 3 of this guidebook, Modules Available.

The Earth Studies modules are written by Christopher Schaufele, Nancy Zumoff and other Mathematics faculty with an interest in these projects.

Introductory Material
The modules written for the Introduction include a Note to the Student, an Introductory Module on World Population, an Introduction to Applets, and an Introductory Practice module on United States Population. The Introductory Module is a module to both begin study of environmental issues and to learn how to use the module and tools. The tools are embedded in this module, along with the resulting answers students should obtain from their work. The Introduction to Applets gives step by step directions on using two of the applet tools, the Java Math Pad and Plot-Solve applets. After using these first materials, the Introductory Practice module is the first module for students to choose resources to use to answer questions.
For a further description of these demonstration modules, go to Chapter 3, Earth Math Introductory Module.

The Introduction modules will use the world population data set and embedded applets with displayed solutions to assist in learning to use the materials.

The Introduction Applets takes the user through two sample applets, the JAVA Math Pad and Plot-Solve Applets.

The practice module revisits the population question with United States data and solutions are embedded for immediate feedback.

Course Materials

Introduction

The Earth Math course materials are written for two algebra levels, *Algebra with Equations* and *Algebra with Functions*. The first is a simpler level intended for the beginning student who has not been introduced to the function notation and is familiar with linear equations. The second is intended for those students who have some experience with function notation and the concept of a function.

The *Pre-Calculus* level is intended for students who are investigating exponential, quadratic, and trigonometric functions as well as linear concepts. The *Calculus* level extends to earlier studies to higher mathematics with model choice and multiple levels of problems. These two levels of study could include the earlier course material as a preliminary study or review of basic mathematical conceptual understanding. The earlier studies allow the student to revisit a linear model study, for example, with greater understanding of needing a more complex model for better prediction.

The environmental issues selected are written for an inquiry based approach to a problem using real data and probing questions with predictions. Students answer questions about implication and reflection of thought, while using mathematical concepts and tools to find results.

A description of each of the levels of study continues below.
Algebra with Equations
The Earth Studies in this level of study are appropriate for Beginning and Intermediate Algebra students (Algebra I and Algebra II). This level is also appropriate for developmental students in college, who are learning fundamental skills in Algebra.

This level student should have a beginning understanding of the use of variables and entering data, with some comprehension of the use of rates. These studies are designed to introduce the student to real application of the mathematics they are learning in the classroom, with an assumption of a very elementary algebra skill level.

The studies available are listed on the course page, as shown below.

Notice the links to the homepage buttons for easy access to course materials. These materials are also useful for review and further study in higher level courses. A more complete description of the studies can be found in Chapter 3, Modules Available.

Algebra with Functions
The Earth Studies in this section are for students in a regular Intermediate and College Algebra courses, at a level traditionally taught as a first course of algebra in college. These studies are not meant for developmental students, but for those beyond the fundamental skill level.
The studies included with this section emphasize the use of models, both linear and curvilinear. Some of the studies have several parts, which can be done individually or as an ongoing project. Each part is designed so it can be done without the other parts, allowing you to pick and choose the modules and data you wish to use.

The modules at this level are appropriate for students with a basic understanding of the use of linear functions and rate of change. Students at this level will work with models of the basic function types (linear, exponential, etc.).

Pre-Calculus

Students using the Pre-Calculus Earth Studies will be using exponential modeling. It is assumed the students using the Pre-Calculus studies are going to continue into the study of Calculus in subsequent courses and therefore Trigonometric modeling skills are included in this level of study.

Pre-Calculus studies use some of the same data sets and questions as the algebra with functions, yet the studies will use models that emphasize exponential models, rather than linear models.

Students at this level can benefit from the review topics that are embedded in each module, particularly those who have a time gap since their last math course.
Calculus
Students using the modules in this level of study are assumed to have the fundamental algebraic and trigonometric skills necessary for the successful study of Calculus. Students need familiarity with all of the basic function types, rate of change, integrals and trigonometric functions.

Students using the Calculus studies will be working with modeling to calculate rates of change and using integrals.

The topics are different from the earlier levels and calculus students can benefit from using the other studies. The studies allow students to revisit the study using their calculus skills and to review some of the topics they might have forgotten.

Students using the Calculus level of study may also use Earth Studies from the lower levels, yet emphasize a view of the data from the Calculus level and revisit the other studies using their Calculus skills.

Navajo Studies
The Navajo Studies modules are written in a related study, designed for use in the Dine Philosophy of Education, as part of the Navajo Nation Rural Systemic Initiative (NN-RSI). The Earth Studies modules are available in the four components of the Dine Philosophy and include references to study on the Navajo Nation. For example, the population module is extended to include a study of the population in several Chapters of the Navajo Nation. Other modules are written to specifically address issues relevant to life on the Navajo Nation, such as Uranium mining and other health issues.

A listing of the modules available for use with Navajo students is given on this home page, as well as information about this project extension for Navajo Nation students.

The Navajo Nation Rural Systemic Initiative Math and Science projects are supported by the National Science Foundation and Dine College.
The Navajo Nation Rural Systemic Initiative

and

The Earth Math Projects

The NNRDI is supported by the National Science Foundation. 
Principal Investigator: Calvin White (ckalvinwhite@navajo.org)
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The EM Projects are supported by the National Science Foundation and the U.S. Dept. of Ed., EIPSE.
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Web Design: Sheree Tso-Varela

Equation Notation:

Demo Module
Practice Module
Streamflow
Population

A listing of each of the modules for use with Native American students is given after the project information.
The modules are essentially the same data sets and questions in the other Earth Math modules, yet are written in the Dine philosophy of education.
They include natural resource consumption, emissions, population, and other topics with an emphasis on reservation concerns.

Evaluation Material

An evaluation of the Earth Math projects is given on this page. This evaluation summary includes the evaluation of the test hypotheses and the major conclusions of the projects. An evaluation summary for the Earth Algebra, Earth Math, Earth Math Phase 3, and other evaluation information is given on this page.

There is a strong indication in each evaluation that the Earth Studies classes show higher success rates than traditional courses, including minority students. For more information go to the Evaluation Material on the website.
Center Tab Button Links

Links to additional web site information is available through these buttons. Current information about the site and materials can be accessed by clicking on the following buttons from the homepage or course module pages.

Announcements

Current announcements about pending presentations and workshops, such as Earth Math Workshop Reunions, will be available through the announcement button. Additional information about meetings and calendar events, as well as project activities will be found through this link.

Bulletin Board

A bulletin board for users of the website and materials will be available to instructors through this button. Questions to other users or project team members can be posted on the bulletin board for response and discussion.

Related Projects

Projects with similar materials or environmental science mathematics will be linked through this button. Projects that have useful materials for the instructor or student using Earth Math Projects will find this button link beneficial. Explore the links provided to see other projects with materials to use.

Current links to Environmental Mathematics (Dr. Ben Fusaro, Florida State Univ.), Mathematics for Human Survival (Dr. Patricia Kenschat, Monclair State Univ.), Workshop Pre-Calculus (Dr. Nancy Baxter-Hastings and Dr. Allan Rossman, Dickinson College), Center for Science Education (Center at Portland State Univ.), Institute for Study in Educational Mathematics, Quantitative Environmental Learning Project, and Kennesaw State University Transitions Project are some of the current projects linked to this site. If you know of great sites or materials that should be available to the users of the Earth Math materials, please let the site hosts know.

References

Links to appropriate data sites and reference materials to use with the Earth Math Projects are available from this button.

FAQ

Answers to the Frequently Asked Questions about the Earth Math Studies and Earth Math web site are available from this link.
Learning to Use Earth Math Modules

This section of the guidebook gives you directions on how to get started and help you prepare for the classes in which you will use the Earth Math modules.

FAQ  How do I get started?

Information for Instructors

Starting the Modules ...................................... 2-1
Web Site Windows ........................................ 2-2
Using the Review Topics ............................... 2-5
Using the Tool Chest ................................. 2-8

Once you have a solid understanding of the components of the Earth Math materials and using the applets, you will continue to have new materials made available to you as the site evolves, and be able to choose the materials and techniques that are suited to your course. Tailoring the Earth Math materials to your courses will enable you to enhance your student’s understanding of the application of the mathematics you teach.

As you use the guidebook, make notes in the pages of this handbook so you have a ready reference for future use of the modules. You should jot down ideas that are discussed and ways you think you may integrate the modules into your class.
Starting the Modules

Standard Formatting of the Earth Math Modules

Introduction
A standard format and methods are used in the analysis of the data and solution of problems that are presented in Earth Studies. The Studies will all be presented in a somewhat structured format that is demonstrated in the Introductory Module described in depth in Chapter Three. The Earth Studies Introductory Module demonstrates the basic format and how to integrate the website materials. We recommend you use the introduction modules as a starting point for students (and instructors) to learn to use the Earth Studies.

The standard format should make the implementation of any of the modules easy – use all or any part of a module, along with the appropriate applets and review. Click on the Course level page and module name. The page opened will be written in the standard format, with components as follows.

Study Components
Each Study will have three components:

- the text, which will provide information about a particular issue and ask questions;
- the Tool Chest, which will contain the applets to perform the mathematical operations;
- the journal, which will be the place where students can record their thoughts, answers, and mathematical solutions.

The text will be presented in four sections:

- The Comprehension section will provide information about the issue and ask pertinent questions for students to think about and express their own ideas regarding the issue.
- The Acquisition section will provide more detailed information, relevant data, or direct students to a website to acquire more information or data.
- The Application section will ask specific questions which will require mathematical analysis that will lead to a mathematical model. The model should be able to provide answers to questions raised regarding the issue. It is in this section that the applets will be used.
- The Reflection section will ask questions regarding the reasonableness or validity of the student’s model.
The Tool Chest will be the place where students can access the applets that will perform the mathematics. Here students can enter their data or other quantitative information and use the applets to analyze data and construct mathematical models. The results should answer the questions involved in the issue. A complete listing of the Tool Chest Applets can be found in Chapter 3, Tool Chest.

**Journal**

The journal will be the place students record their answers to questions from the text and include the explanations of the mathematics they used. Student’s journals should be written as a file in the word processing package on their computer; this will be the document they turn in to you as instructor.

Instructions on selection highlighting, copy and paste of selected text and saving the journal files should be given to students as they begin Earth Studies. It is appropriate to do this while students work with the demonstration modules or the first module you use, if you wish students to use a journal.

### Web Site Windows

**Using the Website**

Go to the website address: [http://earthmath.kennesaw.edu](http://earthmath.kennesaw.edu) to the site homepage. All Earth Math materials available can be accessed through this address. At the Earth Math homepage, click on the study level and scroll down to the module desired and choose the study by clicking on the name. This will automatically open the beginning page of the study and you are ready to begin. You can see this is the same as the previous image, in the CD-Rom instructions above.
Using the CD-Rom

The CD-Rom is provided for you to have an alternative to the website, to have a backup if you are having problems with internet access. Contact a project team member to request a CD-Rom version of Earth Math.

Insert the disk in the drive and open My Computer. Double-click the CD-Rom drive (usually D:) and you will see the contents of the CD.

Double-click on index and it will open the first page of Earth Math Study materials.

Once into the homepage, scroll down the page and click on the study that you want to use and it will link automatically to all windows and materials that are associated with that module.

These materials are the same as those on the website and can be used when no internet access is available. One caution – remember you will not have access to the websites that are linked to the CD if you don’t have internet access, but much of the material can be used – including the applets.
Using the Windows

If a student clicks on Algebra with Equations they will bring up the course module listing for this level. Notice the menu map at the top of this window, which allows the student to go back to the homepage. They can then click on the Predicting Streamflow Module, to start from the beginning, or click on a particular part of the module to pick up where they left off.

A listing of the available modules is included on the course level main page.

Each link is designed to go directly to that module or part of the module.

Notice the link to the Home page in blue, as well as the standard menu links on the left of the page.

You can access any of the materials from these buttons.

If the student clicks on the Predicting Streamflow Part 1 Temperature link, they will bring up the Streamflow Temperature page, as shown below.

The student then will click on the first link, Part 1: Temperature, to begin work in this module.

They may also access the other parts from here. Notice they can also go back to the Streamflow course main page, the Algebra with Equations page or Home page from the top menu. They may also access the other parts of the Streamflow module from this top menu.

The student then will begin work on Part 1: Temperature, beginning with the Comprehension questions, including the use of the applets.

Streamflow Temperature

Part 1: Streamflow Prediction

In this module we will construct a model for average temperatures for spring through fall.

Comprehension

Observation of temperature trends; visualization of method of study.

1. What kind of function would most accurately describe temperature during the months from April through September?
2. Would the function be different for different parts of the world?
Review Topics

The topics provided are for the review of the mathematical concepts and terms included in the study. These concepts are assumed prerequisite concepts and skills required for the successful completion of the study. Students may access these topics to find general information and formulas for working with the data. Interactive applets are included for students to explore the impact of changes on symbolic expressions, graphs, etc. Example problems and solutions are included to assist the student’s review of prerequisite material.

The review topics are listed on the left at the beginning of a module study, with the Tool Chest. Only those topics necessary for successfully completing that particular module of study are included. For example, when opening the Algebra with Functions Passenger Car Gasoline Consumption module, use the scroll bar to follow down the left side of the display which will reveal the following list of Menu Topics for this module.

When a student clicks on the topic name, the topic opens in a new window, to be used along with the study module. For example, if the Linear Functions topic is opened, the following window includes embedded applets and complete examples.
Chapter 2 Learning to Use the Earth Math Modules: Using the Review Topics 2-6

Home / Review Topics

Linear Functions

Definitions

A linear function has the form \( f(x) = mx + b \) for some constants \( m \) and \( b \). The graph is a line with slope \( m \) and \( y \)-intercept \((0,b)\). Its domain is all real numbers since any real number can be substituted for \( x \). For any non-horizontal line, the range is also all real numbers.

Any linear function has a constant rate of increase or decrease. This constant rate of change is the slope of the line and is represented by \( m \) in the equation. Use the following interactive example to explore how the graph of the line \( y = mx + b \) changes as the values of the coefficients \( m \) and \( b \) change. Note any useful observations in your journal.

Slope of a Line

The slope of a line is the constant rate of change of a linear function. It can be thought of as the ratio of the vertical change to the horizontal change between two points on the graph of a line. If the two points are \((x_1, y_1)\) and \((x_2, y_2)\), then the vertical change is \( y_2 - y_1 \) and the horizontal change is \( x_2 - x_1 \). Hence we can use the formula

\[
\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}
\]

to determine the slope of a line if we know two points on that line. See Figure 1 below.

Review Topics may be accessed from the standard menu on the left of the page by clicking on the button link under Instructor’s Materials. The student may select to review prerequisite material at a point that is appropriate in order to work successfully on the Earth Math study.
Review Topics

The Review Topics button is a link to prerequisite review that students can use at any time. Embedded within the modules of study are topics given in the left menu of the page, appropriate for each study. Students can return to the homepage to use a topic they’ve forgotten at any time. Available topics are listed and a description of each follows.

Available Review Topics

**ALGEBRA TOPICS**

- Domain and Range
- Solving Equations
- Solving Equations Graphically
- Linear Equations
- Slope of a Line
- Linear Functions
  1. Slope
  2. Intercepts
- Solving Quadratic Equations
- Quadratic Functions
- Vertex of a Parabola
- Rational Functions
- Exponential Functions
- Exponential Regression
- Exponential Equations
- Composite Functions

**Precalculus Topics**

- Trigonometric Functions: Sine & Cosine
- Period, Phase Shift and Amplitude of Trigonometric Functions
- Trigonometric Equations
- Law of Cosines

**Calculus Topics**

- Rate of Change
- Derivatives
- Applications to Economics
  1. Concepts From Economics
  2. Cost, Revenue and Profit
  3. Marginal Quantities
  4. Supply, Demand and Equilibrium

For example, if a student clicks on the linear functions topic, a window begins with the definition and an applet for reviewing the visual concepts. The general formula for slope and intercept are defined and include examples for the student to review problems appropriate for the topic and level.
The applets are embedded in the Introductory Module and some Review Topics for ease of use but may be accessed from the standard toolbar on the left in any window. Students can choose to follow the study step-by-step, or you may have them do selected parts of the study. You can also assign additional problems for exploration using the applets.

Internet data sites are linked within the study, marked by the blue highlight. Students can click on these links to obtain more data and information.

**Using an Applet**

The Tool Chest is a set of mathematical tools for students to use in their study. Each applet has a window with specific directions for use below the applet window. By clicking on the applet name in the tool chest the student brings up the applet and directions for use. Applets accessed by using a study or review topic that has the applet embedded, do not contain the directions listed with the applet necessarily. Most of the directions for the applets are self-explanatory and there are groups of applets that function in the same manner, with the various models.

To begin to use an applet, open the applet from the Tool Chest, for example the Linear Regression Applet. Notice the window frames for entering the x and y coordinates of the data set, the CLEAR, PLOT and ANALYZE buttons for use. Enter the data set first, than click on PLOT. The applets will open in another window so the window may be resized and used with the module window.

You will need to enter the X coordinates and then the Y coordinates in the Data window.

You then click PLOT and the coordinates are plotted in the top right window.

When you click ANALYZE the ‘best fit’ linear regression model and the resulting function is given in the lower results window.

You may clear the data to enter a new data set by using the clear button.

Students can practice entering data and using the applet buttons, noting the resulting change from additional or new information.

You will notice the graph displayed in the window on the top right and the model in the RESULTS window.
The linear regression applet will plot the data points on the coordinate graph of the applet. Students will see the data points plot in the applet.

This applet result is from the Introductory module.

Click ANALYZE and the best fit regression model for the entered data set will be displayed along with the data set, with the symbolic model and error in the RESULTS window, as shown below.

The linear regression applet will plot the regression line over the data points. Students will find the data model and error in the lower window.
Tool Chest

The Tool Chest is a link to applets that students can use at any time. Embedded within the modules of study and review topics are the links to these applets, given in the left menu of the page. Students can return to the homepage to use an applet to graph or calculate a value at any time. Available applets are listed and a description of each follows. A complete set of directions for each applet are given in Chapter 3, Applets.

Available Applets

COMPUTATION APPLETS

- Exponential Regression
- Linear Regression
- Math Pad
- Plot Solve
- Quadratic Regression

INSTRUCTIONAL APPLETS

- Defining Trig. Functions: Aine & Cosine
- Derivative and Tangent Line
- Exponential Functions
- Function and Derivative
- Graph of Cosine
- Graph of Sine
- Line Slope
- Linear Functions
- Quadratic Functions Standard Form
- Quadratic Functions Vertex
- Radians and Unit Circle
- Riemann Sums

EXPERIMENTATION APPLETS

- Fitting Cosine
- Fitting Exponential Functions
- Fitting Linear Functions
- Fitting Quadratic Functions
- Fitting Sine Functions

For example, if a student clicks on the linear regression applet, a window for calculating the linear regression from a data set is opened. The student can then enter data and plot the data points, then calculate the linear regression equation. The description of each applet is given in the section on applets.

Help assistance is provided for each applet of the Tool Chest. Most applets are self explanatory, with buttons and clear displays. There are several notes regarding the use of the Math Pad and Plot Solve to consider.
Applet Descriptions

The descriptions below briefly identify the function of the applets available. The list follows the same sequence as the website TOOL CHEST.

Computation Applets

- Exponential Regression

**EXPONENTIAL REGRESSION APPLET**

This applet has two functions: plotting and finding the exponential function which best approximates the user supplied data. Using an appropriate data set, this applet plots and data and will find the best fit exponential function for that data. The best fit model is displayed on the graph with the data set for comparison. The symbolic model and error is displayed in a results window.

- Linear Regression

**LINEAR REGRESSION APPLET**

This applet has two functions: plotting and finding the linear function which best approximates the user supplied data. Entering an appropriate data set, the user plots the data and finds the best fit linear function. The function graph is plotted with the data set. The symbolic model and associated error is displayed in the results.

- Math Pad

**JAVA Math Pad**

This applet allows the user to evaluate mathematical expressions and functions. A mathematical expression is made of numbers, variables, built-in and user defined functions together with operations. It has an open screen and several buttons.

For further information, click on the scroll down menu, under "Help Topics" to get more detailed help on how to use this applet.

**You need to enter expressions including all math symbols (, /, ^, etc.) in order to use the math pad. For example:**

Enter function f 1(x) = 2*x Enter f1(3) Gives answer 6.0

This will allow students to define up to ten functions.
• **Plot Solve**

**Plotting Applet**

The purpose of this applet is to plot functions. It can plot up to 10 different functions at the same time. It can also be used to solve equations numerically. Changes in window parameters and function information, using zoom and trace buttons, can allow the user to define and view different functions on the same graph.

• **Quadratic Regression**

**QUADRATIC REGRESSION APPLET**

This applet has two functions: plotting and finding the quadratic function which best approximates the user supplied data. Entering an appropriate data set, the user plots the data and finds the best fit quadratic function. The function graph is plotted with the data set. The symbolic model and associated error is displayed in the results.

**Instructional Applets**

• **Defining Trig Functions: Sine & Cosine**

**The Unit Circle and Trigonometric Functions**

The purpose of this applet is to illustrate the relation between the unit circle and how the trigonometric functions are defined. As you drag a point around the unit circle, the applet displays the coordinates and angle direction. Choosing sine or cosine, the function’s value is displayed as the length of the segment on the unit circle, as well as the corresponding angle.

• **Derivative and Tangent Line**

The purpose of this applet is to illustrate the relationship of the slope of the tangent to the graph of $y = f(x)$ at $x = a$, that can be approximated for a certain value of $h$ where the secant line becomes the tangent to the graph of $y = f(x)$ at $x = a$. This applet illustrates the concept that the slope of the secant approaches the slope of the tangent, allowing the user to enter a function, select the points $(a, f(a))$ and $(a+h, f(a+h))$, to change $h$ so that the point $(a+h, f(a+h))$ gets closer to the point $(a, f(a))$.

• **Exponential Functions**

**Role of $a$ and $b$ in $y = ab^x$**

The purpose of this applet is to illustrate how the values of $a$ and $b$ affect the exponential function $y = a b^x$. Changes made to the general form exponential function, using a scroll bar, will display the affect of the coefficient and base on the resulting graph.
• **Function and Derivative**

This applet illustrates the fact that the derivative of a function \( y = f(x) \) at \( x = a \), denoted \( f'(a) \), is the slope of the tangent to the graph of \( y = f(x) \) at \( x = a \). The user can plot a function, and select points on the graph. As the points are selected, the tangent to the graph at the chosen point is drawn, if desired, its slope is displayed and plotted as a new point. If the user selects enough points, the graph of the derivative will become evident.

• **Graph of Cosine**

**Role of \( a, b, k \) and \( c \) in \( y = a + b \cos(k(x - c)) \)**

The purpose of this applet is to illustrate how the values of \( a, b, c \) and \( k \) affect the graph of the cosine function. Plotting an appropriate data set supplied by the user, the cosine function coefficients and constants selected are then entered and tested by graphing with the data set to view the fit of the function.

• **Graph of Sine**

**Role of \( a, b, k \) and \( c \) in \( y = a + b \sin(k(x - c)) \)**

The purpose of this applet is to illustrate how the values of \( a, b, c \) and \( k \) affect the graph of the sine function. Plotting an appropriate data set supplied by the user, the sine function coefficients and constants selected are then entered and tested by graphing with the data set to view the fit of the function.

• **Line Slope**

**Understanding the Slope of a Line**

The purpose of this applet is to understand the notion of "the slope of a line". By selecting two points on the graph, the point coordinates and quotient rise/run is displayed, as well as the triangle symbolizing the relationship. Point positions can be changed and the information is updated.

• **Linear Functions**

**ROLE of \( m \) and \( b \) in \( y = mx + b \)**

The purpose of this applet is to illustrate how the values of \( m \) and \( b \) affect the line \( y = mx + b \). Using a scroll bar to change the slope \( m \) and/or intercept \( b \), the line position moves to reflect those changes.
• **Quadratic Functions Standard Form**

**Role of a, b and c in** $y = a \, x^2 + b \, x + c$

The purpose of this applet is to illustrate how the values of $a$, $b$ and $c$ affect the parabola $y = a \, x^2 + b \, x + c$. Using a scroll bar to change the coefficients and constant, the parabola's position moves to reflect those changes.

• **Quadratic Functions Vertex**

**Role of a, h and k in** $y = a \, (x - h)^2 + k$

The purpose of this applet is to illustrate how the values of $a$, $h$ and $k$ affect the parabola $y = a \, (x - h)^2 + k$. Using a scroll bar to change the coefficients and constant in the graphing form of a quadratic, the parabola's position moves to reflect those changes.

• **Radians and Unit Circle**

**The Unit Circle and Angles**

The purpose of this applet is to illustrate the relation between a point on the unit circle, its coordinates and the angle between the x-axis and the line through the origin and the point. By dragging a point around the unit circle, the coordinates of the point and the angle values will be displayed in either degrees or radians.

• **Riemann Sums**

**Sums and Approximating Areas**

This applet allows you to experiment with Riemann sums and approximating the area between the x-axis, the graph of $y = f(x)$, the vertical lines $x = a$ and $x = b$. The user specifies what kind of Riemann sum is to be used, the applet draws the graph of the function, the rectangles corresponding to the partition, and the kind of Riemann sum being used.

**Experimentation Applets**

• **Fitting Cosine**

**FITTING TRIGONOMETRIC (COSINE FUNCTION) DATA APPLET**

This applet has two functions: First, it can be used to plot user supplied data. It can also be used to test if a user supplied trigonometric (cosine) function (a function of the form $y = a + b \, \cos(k \, (x - c))$) fits the given data by plotting the function. Plotting an appropriate data set supplied by the user, the cosine function coefficients selected are then entered and tested by graphing with the data set to view the fit of the function.
Fitting Exponential Functions

Fitting Exponential Data Applet

This applet has two functions: First, it can be used to plot user supplied data. It can also be used to test if a user supplied exponential function (a function of the form \( y = a \ (b^x) \) ) fits the given data by plotting the function. Plotting an appropriate data set supplied by the user, the exponential function coefficients selected are then entered and tested by graphing with the data set to view the fit of the function.

Fitting Linear Functions

Fitting Linear Data Applet

This applet has two functions: First, it can be used to plot user supplied data. It can also be used to test if a user supplied linear function (a function of the form \( y = m \ x + b \) ) fits the given data by plotting the function. Plotting an appropriate data set supplied by the user, the linear function coefficient and constant selected are then entered and tested by graphing with the data set to view the fit of the function.

Fitting Quadratic Functions

Fitting Quadratic Data Applet

This applet has two functions: First, it can be used to plot user supplied data. It can also be used to test if a user supplied quadratic function (a function of the form \( y = a \ x^2 + b \ x + c \) ) fits the given data by plotting the function. Plotting an appropriate data set supplied by the user, the quadratic function coefficients and constant selected are then entered and tested by graphing with the data set to view the fit of the function.

Fitting Sine Functions

Fitting Trigonometric (Sine Function) Data Applet

This applet has two functions: First, it can be used to plot user supplied data. It can also be used to test if a user supplied trigonometric (sine) function (a function of the form \( y = a + b \ \sin(k \ (x - c)) \) ) fits the given data by plotting the function. Plotting an appropriate data set supplied by the user, the sine function coefficients selected are then entered and tested by graphing with the data set to view the fit of the function.
This section of the guidebook gives you directions on how to get started and help you prepare for the classes in which you will use the Earth Math modules.

This last chapter of the guide will assist you in understanding how to begin working with the Earth Math Studies. The Introductory Module material can be used to instruct students in the use of the modules, integrating the applet tools and review topics. The Introduction to Applets contains step by step instructions in the use of two applets.
Introductory Material

To The Student

A Note to the Student
This note acknowledges a common student attitude toward the utility of mathematics and application to their lives, due to the lack of meaningful experience. The contrived application does not provide the student motivation to learn and apply mathematical concepts. Note discussion of the design of the modules and tools, the application to everyday problems and issues. The explanation of mathematical study will assist the instructor in encouraging a desire to provide study beyond the traditional problems.

Introductory Module

World Population

In this module, the student is introduced to the format and methods that will be used in the analysis of data and solution of problems that are presented in Earth Studies. The Studies will all be presented in a somewhat structured format that will be outlined in this module. Applets, or little programs, designed to be used with each Study; these will perform the mathematical operations needed to analyze data and build mathematical models. Also, links are included throughout each study that will take students to explanations of mathematical terms and methods. Here you will be guided through a study on population that will illustrate the most common methods used in all of the Earth Math Studies and will also illustrate the use of the applets. This demonstration module can be used as a preliminary study to learn the use of the modules and tools provided in the Earth Studies. First, we outline the general structure for all the Studies.

Each Study will have three components:

- **Text** - which will provide information about a particular issue and ask questions;
- **Menu** - lists in two parts:
  - **Tool Chest** - contain the applets to perform the mathematical operations
  - **Review Topics** - contain links to explanations of relevant mathematical topics and examples;
- **Journal** - which will be the place where students can record thoughts, answers, and mathematical solutions. This is kept on a word processor and should be open as students record the answers to questions.

The Text will be presented in four sections:

- **Comprehension** section - will provide information about the issue and ask pertinent questions for students to think about and express their own ideas regarding the issue.
- **Acquisition** section - will provide more detailed information, relevant data, or direct students to a website to acquire more information or data.
The Application section - will ask specific questions which will require mathematical analysis that will lead to a mathematical model. The model should be able to provide answers to questions raised regarding the issue. It is in this section that the applets will be used.

The Reflection section - will ask questions regarding the reasonableness or validity of a student’s model.

The Tool Chest will be the place where students can access the applets that will perform the mathematics. Here they can enter data or other quantitative information and use the applets to analyze data and construct mathematical models. The results should answer the questions involved in the issue.

The Journal will be the place students record their answers to questions from the text and include the explanations of the mathematics used. The journal should be written as a file in the word processing package on the computer; this will be the document turned in to you as instructor.

Your First Module

EARTH STUDY: WORLD POPULATION

Comprehension

In this section, questions are posed that are designed to stimulate thoughts and observations regarding population increase/decrease, reasons for studying population change, and methods that might be used. At this point, students should open a blank document in their word processor and record the answers to these questions. Copy each question into the journal (Ctrl-C to copy and Ctrl-V to paste) and type their response below the question in a different color or font. Be sure to remind them to write in complete sentences and express their ideas so that others can understand. Students should save their work and keep this window open throughout this study so they can easily record other questions and answers. Each of the applets is embedded in this first module and an additional applet display follows with the correct result. The applet tools are embedded in this first Introductory Module only.

Questions
A. Do you think that the population of the World is increasing or decreasing?
B. What have you noticed recently that led you to your answer to A?
C. How do you think population change in the world might influence your life?
D. How do you think mathematics can be used to study population change?
E. What are some reasons for studying population change?
Acquisition

In this section, students will be given information about population increase, real data is provided, and links to websites that contain more information and updated data are also included.

Estimates of world population before the twentieth century vary widely, but most sources put the number of people in the world in 1750 at about 750 million. By mid-nineteenth century the population reached 1 billion and until 1930 the growth was never more than 1%; however, since 1950 the increase has never fallen below 1.6%. Current patterns of population growth and the accompanying changes in consumption are placing increasing stresses on ecosystems through environmental degradation, deforestation, loss of biological diversity, over harvesting, and accumulation of toxic wastes.

The Table below gives the United Nations estimates of the world population (in billions) every five years from 1950-1995. Students producing a model for prediction of population will use this table of information.

|------|------|------|------|------|------|------|------|------|------|------|

World Watch 1996

In this section, objectives and assumptions are stated; these will determine the mathematical model for population. Students will use relevant mathematical tools that are provided, the applets, to perform an analysis on the data and build an appropriate model. Students will develop a model for world measure of population growth. Applets can be accessed by clicking “Tool Chest” on the Menu to the left of the screen in the demonstration module.

Instructors and students should follow the steps outlined after each question to familiarize themselves with the techniques used and the applets available. Record answers in the student journal.

The objectives and assumptions are listed below.

Objectives
To determine:
1. a linear model for world population;
2. the annual rate of change of world population.

Assumption
The current trends for world population continue.
Application

Linear Models
In this part, students will find a linear function to approximate data for world population and use this to
- estimate annual population growth,
- estimate the population for years other than those in the data set, and
- forecast future population size.
(Note the link to a Topic for review, if the student needs to review linear functions.)
(Students should round off to three places for this work.)

Problem Set
The applets needed to complete this problem are provided in the text for this demonstration module. Usually, students will need to follow the following steps.
- Scroll down to “Tool Chest” on the applet menu to the left of the screen.
- Click “linear regression” (since we are looking for a linear function).

The linear regression applet will open in a separate screen. Students will enter the data in the applet.

1. Plot the points corresponding to the data in Table 1. The first coordinate is year; denote this by t with t = 0 in year 2000. The second coordinate is population in billions.

   Enter the data provided in Table 1; enter the t value for each year (t = -10 for 1990, t = -5 for 1995, etc.) in the “x” column, then enter the population figures in the “y” column.

   Click “Plot”.

The linear regression applet will plot their data points on the coordinate graph of the applet. Students will see the data points plot in the applet.
2. Determine the linear regression function that best fits these data; call this function \( P(t) \). Graph the function \( P(t) \) on the same coordinate system as the plot of the data points. Click “Analyze”. The screen shown below should appear.

Students should obtain the linear function \( P(t) = 0.072t + 5.952 \). Here, the variable \( x \) has been replaced by \( t \), and the variable \( y \) has been replaced by the function notation \( P(t) \); coefficients are rounded to three places according to instructions. The graph is shown on the applet screen.

3. What is the slope of \( P(t) \)? Students can give a verbal interpretation of the answer; make sure they identify the units clearly.

The slope is \( m = 0.072 \); this means that the world population is growing at the rate of 0.072 billion (72,000,000) people each year. (There is not an applet for this!)

Use the function \( P(t) \) to answer the following questions.

4. What is the annual population growth?

The annual population growth is 72,000,000 people (see #3)

The next computations can be done using the "Java Math Pad" applet in the Tools Menu.

Usually, students will need to follow these steps.

Scroll down to “Tool Chest” on the menu to the left of the screen.

Click “Java Math Pad”. This applet performs mathematical evaluations.

5. How much will the population grow in 10 years? Six months? One week?

Type 10*72000000 then press enter. Your answer will appear in scientific notation, 7.2E8 which is 720,000,000 people. Or, you can type 10*.072, press enter. Here your answer will be .72 billion people.
Type \( \times 72000000 \) then press enter.
Type \( \times (1/52) \times 72000000 \) then press enter.

In 10 years the population will grow by \( \times 72000000 = 720000000 \) people; in 6 months, \( 0.5 \times 72000000 = 36000000 \) people; in one week, \( (1/52) \times 72000000 = 1384615 \) people.

The screen shown below should appear. The Java Math Pad applet will perform mathematical evaluations. Students will find the value of basic computation and evaluation of expressions with this applet.


   In the year 2005, \( t = 5 \). You can continue using the "Java Math Pad".

   Type in \( P(t) = 0.072 \times t + 5.952 \), then "Enter". Next type \( P(5) \), then "Enter". You should see the value of the function \( P \) at \( t = 5 \):

   \[
P(5) = 6.312 \text{ (rounded to 3 decimal places)}.
   \]

   In the year 2005, there will be approximately 6.312 billion \((6,312,000,000) \) people in the World.

7. Predict when the population will reach \( 8,000,000,000 \).

   To make this prediction, you can solve the linear equation

   \[
P(t) = 8.
   \]

   Students can do this prediction by hand algebraically fairly easily but we illustrate an applet that will instead do this graphically. This will make a student's work easier for more complicated equations. The graphical solution will be the intersection of the graphs of \( P(t) \) and the horizontal line \( y = 8 \).

   Scroll down the menu to the "Tool Chest", then click Plot-Solve. (First, read the "help" section.) Enter the first function \( P(t) \), then enter the second function as the constant 8. Click "plot". Then click on the screen to see a red point on the graph of one of the functions. Click on "right" or "left" to move the point to the intersection of the two graphs. You may want to zoom in for more accuracy. The coordinates of the point can be read on the screen.
The screen shown below should appear.

The Plot-Solve applet will plot functions, given window parameters and the expression that defines the function. Students will find the applet will plot several functions on the same coordinate plane and can use zoom and trace to find the intersections of the given functions.

In the year 2028 (t = 28.4), there will be approximately 8 billion people in the World.

Reflection

In this section, students will examine their model, solutions, and implications. Students will think about the validity and reasonableness of their answers. How closely does the function fit the data? Relevant questions are provided although students may raise questions of their own. Record student answers to these questions and thoughts of their own in the student journal. Students return their journals to you upon completion of the module.

Questions

A. Do you think that a linear function is good to use for this study? Are there other functions that you think might provide a better model? Why?

B. How long do you think this model will be accurate; i.e., what is a reasonable domain for the function?

C. How do you think the predicted increase in world population might affect future life in the World? In the United States? In your home town?
MODULeS AAvAILABLE

Introductory Studies

Tutorial Demonstration Studies
The following demonstration studies are available to both the instructor and the student for use while learning to use the Earth Studies modules. A demonstration study, a practice study, and an applet demonstration are included in the learning modules.

- **To the Student**
  This first note to the student is a discussion of the student perception and real application of mathematics to modeling. It includes the relevance of the topic and important issues presented in the modules.

- **Introductory Module**
  This sample study on World population is designed to assist the instructor in working with the Earth Math materials, both for their own reference and to use with the students as a first, directed and step-by-step introduction to the format of the modules. All components of the study have directions for using the data and applets. A practice study is included for use after students complete the demo module.

- **Introduction to Applets**
  This module is a demonstration in the use of the applets. Specific directions to the use of the applets in the Tool Chest menu are given, using the Java Math Pad and the Plot-Solve applets as an example.

- **Introductory Practice**
  This sample study on United States population is designed to use with the students as a first module practice, directed and step-by-step to use the applets and follow the format of the modules. All components of the study have directions for using the data and applets. A practice study is included for use after students complete the demo module.

Earth Studies

Earth studies are provided for different levels of mathematical study. Levels of study are for foundation mathematics Algebra with Equations, intermediate level Algebra with Functions, Pre-calculus and Calculus.

**Levels Available**

**Algebra with Equations**

These Modules are appropriate for foundation mathematics: introductory and intermediate level algebra. They emphasize linear, quadratic, piecewise, and prediction from these models.

**Fuel Economy**
This is a study of the fuel economy rates for two different vehicles and the cost of fuel for each over a given time period. A comparison of the miles driven and fuel used for each vehicle leads to the cost analysis of the vehicle driven.
Chapter 3 Instruction With Earth Math: Modules Available 3-9

Predicting Streamflow  This study looks at streamflow rates for the foundation algebra level. The three parts together make up a complete study on predicting streamflow. However, each part is designed so that it can be done independently. Relevant answers from preceding parts are included in each part as needed.

**Temperature**  In this module we will construct a model for average temperatures for spring through fall.

**Precipitation**  In this module we will construct a model which gives the rate of precipitation in a certain region of the country.

**Snowmelt**  In this module, we will determine the streamflow for a river. It will first be necessary to find out how much snow contributes to the flow, then the volume of water distributed over the watershed for the river.

Algebra with Functions

These Modules are appropriate for college algebra level mathematics. They emphasize linear, quadratic, piecewise, and prediction from these models.

**Carbon Dioxide Concentration** (linear) The study is in two parts. In the first part you will model atmospheric carbon dioxide concentration. In the second part you will use this model to predict the impact of changes in concentration on global temperature and ocean level.

**Carbon Dioxide Concentration**  This study will model the CO2 concentration with a linear model for the last forty years and predict the future levels.

**Impact: Temperature**  This module examines the possible impact of changes in atmospheric carbon dioxide concentration on the global temperature.

**CO2 Emission from Passenger Vehicles**  This Earth Math Study is written in three parts. The three parts together make up a complete study on predicting carbon dioxide emission from passenger vehicles. However, each part is designed so that it can be done independently. Relevant answers from preceding parts are included in each part as needed.

**Passenger Car Gasoline Consumption**  In this first part you will study the factors that affect passenger car gasoline consumption including the number of vehicles, annual mileage, and average fuel efficiency.

**CO2 Emission from Passenger Cars**  In the second part you will model gasoline consumption for passenger vehicles in the United States and you will use this model to estimate the amount of CO2 emitted by passenger cars.

**Reducing Emissions from Passenger Cars**  In part three you devise a plan to decrease emissions.

**Streamflow**  The three parts together make up a complete study on predicting streamflow. However, each part is designed so that it can be done independently. Relevant answers from preceding parts are included in each part as needed.
**Temperature**  This module is the first of three that are designed to lead to the prediction of streamflow for a river in a particular region.

**Precipitation**  In this module we will construct a model which gives the rate of precipitation in a certain region of the country.

**Snow Melt and Streamflow**  In this module, we will determine the streamflow for a river.

**Population**  This sample study on World population is designed to construct a linear model and predict the world population values.  It explores the patterns in slope and the linear function, as well as the accuracy of prediction.

**Precalculus**

These Modules are appropriate for pre-calculus level mathematics. They emphasize rate of change, exponential, trigonometric functions, and prediction from these models. These studies are designed to cover the same data and questions that the Algebra version does but it assumes the student has completed the foundation courses of Algebra.

**Carbon Dioxide Concentration**  (Exponential) This study is in two parts. In the first part you will model atmospheric carbon dioxide concentration. In the second part you will use this model to predict the impact of changes in concentration on global temperature and ocean level.

- **Concentration**  This study will model the CO2 concentration with a linear model for the last forty years and predict the future levels.

- **Impact**  This module examines the possible impact of changes in atmospheric carbon dioxide concentration on the global temperature.

**Population**  This sample study on World population is designed to construct an exponential model and use rate of change to predict the world population values.

**Streamflow**  The three parts together make up a complete study on predicting streamflow. However, each part is designed so that it can be done independently.

- **Temperature**  This module is the first of three that are designed to lead to the prediction of streamflow for a river in a particular region.  It looks at how temperature and amount of precipitation affect the amount of water in a river.

- **Precipitation**  This module is the second of three that are designed to lead to the prediction of streamflow for a river in a particular region.  A model for average precipitation for the region is constructed using the previous model for temperature.

- **Snow Melt and Streamflow**  This module is the third of three that are designed to lead to the prediction of streamflow for a river in a particular region. Results from the previous parts are combined with area to compute monthly streamflow figures.
The Gas-Mileage Bill and The Arctic National Wildlife Refuge
This module explores the efficiency of automobile gasoline consumption and the feasibility of drilling in the Artic National Wildlife Refuge to relieve this nation’s dependency on foreign oil. Students use earlier explorations on gasoline consumption to extend this into a prediction equation for oil consumption.

Water Consumption in the United States
This module explores the water consumption in the United States and a trend comparison of the population, water withdrawal, and per capita usage.

Calculus
These Modules are appropriate for calculus level mathematics. They emphasize rate of change, integrals, derivatives, demand functions and prediction from these models.

World Coal Supply
In this module we will study availability and world use of coal. In particular, we will determine the current usage rate and how long the coal supply will last at this rate of consumption.

World Grain
The three parts together make up a complete study on world grain supply and demand. However, each part is designed so that it can be done independently. Relevant answers from preceding parts are included in each part as needed.

Grain Supply
This part of the study looks at the area of the land cultivated and the world grain supply yield.

Supply versus Demand
This part looks at the worldwide demand for grain and when the demand will equal the supply.

Per Capita Supply
This part derives a function that describes the per capita grain supply. Students predict how much grain will actually be available per person.

World Population
The world population growth rate is revisited as the rate of change described as a percent of the population. Students see what affect the change in growth rate has on the population and prediction.

Drilling For Dollars
In this study we will examine oil supply and gasoline demand in a small (fictional) community. In connection with this we analyze the economics involved in the sale of oil and the purchase of gasoline. Specifically, we will be concerned with cost, revenue and profit associated with the production and sale of a product.

Cost of oil
This part explores the factors affecting the cost of a barrel of oil. Fixed, variable and marginal costs are included in the discussion to develop a cost function.
**Revenue and profit**  Students examine the revenue and profit for a company and use a cost function to see how the selling price influences the profit.

**Supply curve**  Students examine the relationship between selling price of oil and the amount of oil the company is willing to extract.

**Supply, demand and equilibrium**  Now we assume that currently the market is in equilibrium. You have derived the oil supply curve (step three), and will determine the current quantity of oil demand, the current price, and the demand equation.

**The Gas-Mileage Bill and The Arctic National Wildlife Refuge**

This module explores the efficiency of automobile gasoline consumption and the feasibility of drilling in the Artic National Wildlife Refuge to relieve this nation’s dependency on foreign oil. Students use earlier explorations on gasoline consumption to extend this into a prediction equation for oil consumption.

**Water Consumption in United States**

This module explores the water consumption in the United States and a trend comparison of the population, water withdrawal, and per capita usage.

**A Village and Its Resources**

This study is the investigation of available resources for a growing village. It is divided into three parts: Population; Food, Coal and Electricity; and Water. Models are used to predict the natural and human consumption of resources for the village, as well as when the water source will be totally depleted.

**Other Modules**  The authors have other planned modules that will be available on-line in the future. Keep watching the site for updates.
Review Topics

Menu Topics

The topics provided are for the review of the mathematical concepts and terms included in the study. Students may access these topics to find general information and formulas for working with the data. Interactive, embedded applets are included for students to explore the impact of changes on symbolic expressions, graphs, etc. Example problems and solutions are included to assist the student’s review of prerequisite material.

The Menu Topics are listed on the left at the beginning of a module study near the Acquisition component, with the Tool Chest. Only those topics necessary for successfully completing that particular module of study are included.

All the topics can be accessed from the standard menu on the left of the screen under Instructor’s Material from any screen of the site. Topics accessed this way are outlined in the following levels.

ALGEBRA TOPICS

The topics in this section are appropriate for the review of definitions, formulas, and concepts at the beginning and intermediate algebra levels. Each contains appropriate examples for students to try and use of real data.

- Domain and Range
- Solving Equations
- Solving Equations Graphically
- Linear Equations
- Slope of a Line
- Linear Functions
  1. Slope
  2. Intercepts
- Solving Quadratic Equations
- Quadratic Functions
- Vertex of a Parabola
- Rational Functions
- Exponential Functions
- Exponential Regression
- Exponential Equations
- Composite Functions
Precalculus Topics

The topics in this section are appropriate for the review of definitions, formulas, and concepts at the pre-calculus level, including functions and use. Each contains appropriate examples for students to try and use of real data.

- Trigonometric Functions: Sine & Cosine
- Period, Phase Shift and Amplitude of Trigonometric Functions
- Trigonometric Equations
- Law of Cosines

Calculus Topics

The topics in this section are appropriate for the review of definitions, formulas, and concepts at the calculus level, including functions and use. Each contains appropriate examples for students to try and use of real data.

- Rate of Change
- Derivatives
- Applications to Economics
  1. Concepts From Economics
  2. Cost, Revenue and Profit
  3. Marginal Quantities
  4. Supply, Demand and Equilibrium

Calculus students should also be encouraged to use any of the earlier review topics, as they need them.

Sample Available Topics

The following Menu Topics were selected to demonstrate the topics available on the web site. The topics included for review are numerous and thorough and this is just a sampling of what is available to both the student and instructor. The embedded applets are available in the review topic on the web site.

Demonstration and Practice Modules Topics

Linear functions

Definitions
A linear function has the form $f(x) = mx + b$ for some constants $m$ and $b$. The graph is a line with slope $m$ and $y$-intercept $(0,b)$. Its domain is all real numbers since any real number can be substituted for $x$. For any non-horizontal line, the range is also all real numbers.

Any linear function has a constant rate of increase or decrease. This constant rate of change is the slope of the line and is represented by $m$ in the equation. Use the following interactive example to explore how the graph of the line $y = mx + b$ changes as the values of the coefficients $m$ and $b$ change. Note any useful observations in your journal.
**Slope of a Line**
The slope is the constant rate of change of a linear function. It can be thought of as the ratio of the vertical change to the horizontal change between two points on the graph of a line. If the two points are \((x_1, y_1)\) and \((x_2, y_2)\), then the vertical change is \(y_2 - y_1\) and the horizontal change is \(x_2 - x_1\). Hence we can use the formula \(m = \frac{y_2 - y_1}{x_2 - x_1}\) to determine the slope of a line if we know two points on that line. See Figure 1 below.

**Figure 1**
Explore the following interactive example to see how the slope of a line is computed when two points are known. What observations can you make?

In general, when the slope of a linear function is positive, the function will be increasing. That is, the graph will be a line that rises from left to right. When the slope is negative, the function will be decreasing and the graph will be a line that falls from left to right. If the slope of a linear function is 0, then the function is neither increasing nor decreasing, but is constant. The graph of a constant function is a horizontal line.

Use the next interactive example to see how different values of the slope, \(m\), affect the graph of the linear function \(y = mx + b\).

**Horizontal and Vertical Intercepts**
The points where a line crosses the vertical and horizontal axes are known as the vertical and horizontal intercepts. (These points are often referred to as the x-intercept and the y-intercept.) Given a linear function \(f(x) = mx + b\),

1. The **vertical intercept** (y-intercept) is found by evaluating the function when the input variable, \(x\), is 0 and is always the same as the constant \(b\). It can be thought of as the original value of the function.
2. The **horizontal intercept** (x-intercept) is the value of the variable \(x\) when the function value is 0. It is found by solving the equation \(0 = mx + b\).

**Interactive Example**
Now explore how the values of the y-intercept, \(b\), affect the graph of the linear function \(y = mx + b\). (Applet is embedded here)

**Algebraic Example**
Find the vertical and horizontal intercept of the linear function \(f(x) = -7.2x + 250\).

**Solution**
Since \(f(0) = -7.2(0) + 250 = 250\), the vertical intercept is 250. This means that the graph of the linear function crosses the horizontal axis at the point \((0, 250)\). Also notice that this is the value of \(b\) in the linear function \(f(x) = mx + b\).

To find the horizontal intercept we can replace \(f(x)\) with 0 and solve the linear equation \(0 = -7.2x + 250\). The solution is given below.
Chapter 3 Instruction With Earth Math: Review Topics 3-16

The horizontal intercept is 34.7. This is the point (34.7, 0) on the graph of the linear function. A graph of the linear function is shown in Figure 2.

Linear Regression

When we have a set of data points that appear to lie on a line (or almost on a line), we will find a linear function to model the data. There are many different lines that could be drawn through a set of data points, but we want to find the “best” line. The process used by statisticians and mathematicians to find the “best” line for a set of data is called linear regression and the resulting line is called the regression line. We can use the linear regression tool to find the regression line for a set of data.

Example

Find the linear regression equation for the data given in the following table.

<table>
<thead>
<tr>
<th>x</th>
<th>57</th>
<th>67</th>
<th>77</th>
<th>87</th>
<th>97</th>
</tr>
</thead>
</table>

Solution

We first enter the data in the x and y columns. Then click the Plot button to plot the points and the Analyze button to find the regression line. See Figure 1.

The regression line is $y = 0.222x - 3.046$. (The coefficients have been rounded to three decimal places.)
Solving Equations

Solving Linear Equations
A linear equation has the form \( ax + b = c \), where a, and b are constants and x is the variable. To solve a linear equation, we must find the value of the variable which make the equation true. An algebraic method that can be used to solve linear equations is the following: Replace the equation with a simpler equivalent one and continue this process until a solution is reached. Below are some procedures that result in equivalent equations.
Interchange the two sides of the equation. Simplify either or both sides of the equation by removing parenthesis, adding like terms, etc.
Add or subtract the same expression from both sides of the equation.
Multiply or divide both sides of the equation by the same non-zero expression.

The following examples illustrate these ideas.

Example
Solve the linear equation
\[ 100 = -7.2t + 250 \]
Round the answer to two decimal places if necessary.

Solution
\[
100 = -7.2t + 250
\]
\[-150 = -7.2t \text{ (Subtract 250 from both sides)}
\]
\[
-150 = -7.2t \quad \text{ (Divide both sides by -7.2)}
\]
\[
t = 20.83 \quad \text{ (Interchange the two sides of the equation and round to two decimal places)}
\]

Example
Solve the linear equation
\[ -3.2(0.7x + 12) + 10.5 = -0.8x + 1.6 \]
Round the answer to two decimal places.

Solution
\[-3.2(0.7x + 12) + 10.5 = -0.8x + 1.6 \]
\[-2.24x - 38.4 + 10.5 = -0.8x + 1.6 \text{ (Remove parenthesis on the left)}
\]
\[-2.24x - 27.9 = -0.8x + 1.6 \text{ (Combine like terms on the left)}
\]
\[-1.44x = 29.5 \text{ (Add 0.8x to both sides)}
\]
\[
x = \frac{29.5}{-1.44} \quad \text{ (Divide both sides by -1.44)}
\]
\[
x = -20.49 \quad \text{ (Rounded to two decimal places)}
\]

Solving Quadratic Equations

Factoring Method
A quadratic equation has the form \( ax^2 + bx + c = 0 \).
If the expression \( ax^2 + bx + c \) can be factored, then we can set each factor equal to zero. The following example illustrates the factoring method.

Solve the quadratic equation
\[ 3x^2 - 7x + 2 = 0 \]

Solution
\[ (3x - 1)(x - 2) = 0 \text{ (Factor the expression on the left side)}
\]
\[ 3x - 1 = 0 \text{ or } x - 2 = 0 \text{ (Set each factor equal to zero)}
\]
\[ 3x = 1 \text{ or } x = 2 \text{ (Solve each equation separately)}
\]
\[ x = \frac{1}{3} \text{ or } x = 2 \]
The Quadratic Formula

Since most quadratic expressions cannot be factored, the factoring method is limited in its usefulness. However, any quadratic equation \( ax^2 + bx + c = 0 \) that has real solutions can be solved by the quadratic formula

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.
\]

The quantity \( b^2 - 4ac \) is called the discriminant of the quadratic equation because it determines whether the equation has one, two, or no real solutions: If \( b^2 - 4ac > 0 \), the equation has two real solutions. If \( b^2 - 4ac = 0 \), the equation has exactly one solution. If \( b^2 - 4ac < 0 \), the equation has no real solutions.

**Example**

Use the quadratic formula to find the solutions, if any, of the quadratic equation \(-1.8x^2 - 7.6x + 11.4 = 0\).

**Solution**

Comparing our equation to \( ax^2 + bx + c = 0 \), we see that \( a = -1.8, b = -7.6, \) and \( c = 11.4 \). Substituting these values into the quadratic formula we get

\[
x = \frac{7.6 \pm \sqrt{(-7.6)^2 - 4(-1.8)(11.4)}}{2(-1.8)}.
\]

Evaluating this expression, we find two solutions, \( x = -5.49 \) and \( x = 1.17 \) (rounded to two decimal places).

**Example**

Use the quadratic formula to find the solutions, if any, of the equation \(-16t^2 + 45t + 200 = 160\).

**Solution**

In this case, we must first subtract 160 from both sides so that the equation is in the form \( ax^2 + bx + c = 0 \).

\[-16t^2 + 45t + 200 = 160\]

\[-16t^2 + 45t + 40 = 0\].

We now see that \( a = -16, b = 45 \) and \( c = 40 \). Substituting these values into the quadratic formula, we get,

\[
t = \frac{-45 \pm \sqrt{45^2 - 4(-16)(40)}}{2(-16)}.
\]

Evaluating this expression yields the two solutions \( t = -0.71 \) and \( t = 3.52 \) (rounded to two decimal places).

### Solving Equations Graphically

Almost any equation in one variable can be solved graphically. The strategy is as follows: graph the left hand side and the right hand side of the equation on the same coordinate axes, and then trace the graph to find the point(s) where the two curves intersect. If there is no point of intersection then we say that the equation has no solution. The following example will illustrate this procedure.

**Example**

Find all solutions of the equation \( 0.15x^2 - 6.28x + 37.61 = 60 \). Round your answers to three decimal places.

![Graph of the equation](image-url)
The graph shows that there are two points of intersection. Click on one of the points and zoom in to find the coordinates of that point of intersection. Figure 2 shows that the intersection point on the left has coordinates (-3.3, 60).

We can repeat the process to find the right intersection point. The coordinates are (45.2, 60).

The equation has two solutions, \( x = -3.3 \) and \( x = 45.2 \) (accurate to one decimal place).

**Menu Topics**  Fuel Economy  
**Domain and Range**

The *domain* of a function is the set of all possible input values. If the domain is not specified, we usually take it to be the set of all real numbers for which the function is defined.

The *range* of a function is the set of all outputs or in other words, the set of all possible answers when the domain values are substituted for the input variable. When a function is given by a formula the range is often difficult to determine. If we can graph the function, we can look at its graph to determine the range.

We can think of population as depending on time in years so the independent variable or input is the year and the dependent variable or output is the U.S. population. Since the table gives a unique population for each year, it represents a function. The domain is the set of years (1960, 1970, 1980, 1990) and the range is the set of populations in those years (181 million, 205 million, 228 million, 250 million). Source:

**Example**

**A Function Given by a Table of Values**

The following table gives U.S. population in millions in the indicated year:

<table>
<thead>
<tr>
<th>Year</th>
<th>U.S. Pop (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>181</td>
</tr>
<tr>
<td>1970</td>
<td>205</td>
</tr>
<tr>
<td>1980</td>
<td>228</td>
</tr>
<tr>
<td>1990</td>
<td>250</td>
</tr>
</tbody>
</table>

Example
A Rational Function

Determine the domain of the function
\[ g(x) = \frac{1}{x - 3}. \]

Solution
The function \( g(x) \) is not defined when \( x = 3 \) since division by 0 is not allowed. The only problem in evaluating \( g(x) \) occurs when the denominator equals 0, that is, when \( x = 3 \), so the domain of \( g \) is the set of all real numbers except 3. In set notation we write
\[ \{ x | x \neq 3 \} \]
or in interval notation
\[ (-\infty, 3) \cup (3, \infty). \]

In general, the domain of a rational function is the set of all numbers for which the denominator is not zero. Since \( g(x) = 0 \) for all \( x \), the range is the set of all real numbers except 0.

Example The Square Root Function

A different kind of situation is presented by the square root function \( h(x) = \sqrt{x} \).

No negative numbers can be substituted for \( x \) because you can't take the square root of a negative number and get a real number. But anything else is O.K. to substitute for \( x \), so the domain of \( h \) is the set of all non-negative real numbers,
\[ \{ x | x \geq 0 \} \text{ or } [0, \infty). \]

Since \( h(x) = \sqrt{x} \geq 0 \) for all \( x \), the range is also the set of non-negative real numbers.

Example Linear Functions

Determine the domain and range of the linear function \( y = -3.2x + 14.7 \).

Solution
Since any real number can be substituted for \( x \), the domain consists of all real numbers. A graph of the function in Figure 1 below shows that the range is also the set of all real numbers.

Example

Determine the domain and range of the quadratic function
\[ f(x) = 3x^2 - 5x + 7. \]

Solution
Again, since any real number can be substituted for \( x \), the domain consists of all real numbers. Since the graph of this function is a parabola opening up, it has a minimum value, which is the \( y \)-coordinate of the vertex. Using the vertex formula we find that the vertex is
\[ x = -\frac{b}{2a} = -\frac{5}{2(3)} = \frac{5}{6} \] and
\[ y = f\left(\frac{5}{6}\right) = 3\left(\frac{5}{6}\right)^2 - 5\left(\frac{5}{6}\right) + 7 = \frac{50}{12} = 4.167 \]

So, the range is \( \{ y | y \geq 4.167 \} \).
Example Polynomial Functions
The domain of a polynomial function is always the set of all real numbers. However, the range depends on the particular function, so you should always graph the function to determine the range. As an example, consider the cubic function $h(x) = -x^3 + 2x^2 + 3x - 1$ whose graph is shown in Figure 3.

Example The Sine and Cosine Functions
Recall that the sine and cosine are functions of a real number $\theta$, the radian measure of an angle. Also, given any real number $\theta$, there is an angle with measure $\theta$ radians. So, the domain of these two trigonometric functions is the set of all real numbers. Since the maximum output value is 1 and the minimum output value is $-1$, the range is the interval $[-1, 1]$.

Example A Function Representing a Real-World Situation
There are practical considerations to take into account when the function represents a real problem. Consider the function $J(p) =$ the unemployment rate (percentage of people unemployed) when the gross national product is $p$.

Solution
The input is the gross national product, the monetary value of all goods produced in the country. Thus $p \geq 0$ and the domain of function $J$ is all non-negative real numbers, $\{p | p \geq 0\}$.

The range of the function is the set of real all numbers between 0 and 100 since the unemployment rate is a percentage.

Many other excellent review Menu Topics are available for the instructor and student. These are only a small sample of style and format contained within the Review Topics Menu and within each of the Earth Studies Modules.
Applets

The Tool Chest is a link to applets that students can use at any time. Embedded within the modules of study and review topics are the links to these applets. Students can return to the homepage to use an applet and graph or calculate a value at any time.

Available Applets

COMPUTATION APPLETS

- Exponential Regression
- Linear Regression
- Math Pad
- Plot Solve
- Quadratic Regression

INSTRUCTIONAL APPLETS

- Defining Trig Functions
- Derivative and Tangent Line
- Exponential Functions
- Function and Derivative
- Graph of Cosine
- Graph of Sine
- Line Slope
- Linear Functions
- Quadratic Functions Standard Form
- Quadratic Functions Vertex
- Radians and Unit Circle

EXPERIMENTATION APPLETS

- Fitting Cosine
- Fitting Exponential Functions
- Fitting Linear Functions
- Fitting Quadratic Functions
- Fitting Sine Functions

For example, if a student clicks on the linear regression applet, a window for calculating the linear regression from a data set is opened. The student can then enter data and plot the data points, then calculate the linear regression equation. The description of each applet is given in the section on applets.

Help assistance is provided for each applet of the Tool Chest. Most applets are self explanatory, with buttons and clear displays. There are several notes regarding the use of the Math Pad and Plot Solve to consider.
Applet Descriptions

Computation Applets

- **Exponential Regression**

**EXPONENTIAL REGRESSION APPLET**

This applet has two functions: plotting and finding the exponential function which best approximates the user supplied data. Using an appropriate data set, this applet plots and data and will find the best fit exponential function for that data. The best fit model is displayed on the graph with the data set for comparison. The symbolic model and error is displayed in a results window.

Use it as follows:

1. Put the data to plot and analyze in the Data section. Though the variables default to x (the independent variable) and y (the dependent variable), they can be renamed. Renaming them will cause the graph to be updated. The values are entered one per line.

2. The buttons work as follows: **CLEAR** will clear the data, the plot area and the RESULTS area. **PLOT** will plot the data as points. **ANALYZE** will find the exponential function which best approximates the data, plot it and display its equation in the RESULTS area.

3. The plot area (top, right) will show the plot.

4. The RESULTS area is used to communicate with the user. Errors will be displayed there, for example if the number of values is not the same for both variables. Information will also be displayed there. For example, when the user clicks on **ANALYZE** the equation of the regression function will appear there, as well as the error associated with this function.

- **Linear Regression**

**LINEAR REGRESSION APPLET**

This applet has two functions: plotting and finding the linear function which best approximates the user supplied data. Entering an appropriate data set, the user plots the data and finds the best fit linear function. The function graph is plotted with the data set. The symbolic model and associated error is displayed in the results.

Use it as follows:

1. Put the data to plot and analyze in the Data section. Though the variables default to x (the independent variable) and y (the dependent variable), they can be renamed. Renaming them will cause the graph to be updated. The values are entered one per line.

2. The buttons work as follows: **CLEAR** will clear the data, the plot area and the RESULTS area. **PLOT** will plot the data as points. **ANALYZE** will find the linear function which best approximates the data, plot it and display its equation in the RESULTS area.

3. The plot area (top, right) will show the plot.

4. The RESULTS area is used to communicate with the user. Errors will be displayed there, for example if the number of values is not the same for both variables. Information will also be displayed there. For example, when the user clicks on **ANALYZE**, the equation of the regression function will appear there, as well as the error associated with this function.
• **Math Pad**

**JAVA Math Pad**

This applet allows the user to evaluate mathematical expressions and functions. A mathematical expression is made of numbers, variables, built-in and user defined functions together with operations. It has an open screen and several buttons.

For further information, click on the scroll down menu, under "Help Topics" to get more detailed help on how to use this applet.

**You need to enter expressions including all math symbols ( , /, ^ , etc.) in order to use the math pad.** For example:

Enter function \( f_1(x) = 2x \) Enter \( f1(3) \) Gives answer 6.0

This will allow students to define up to ten functions.

• **Plot Solve**

**Plotting Applet**

The purpose of this applet is to plot functions. It can plot up to 10 different functions at the same time. It can also be used to solve equations numerically. Changes in window parameters and function information, using zoom and trace buttons, can allow the user to define and view different functions on the same graph.

It functions as follows:

1. The **Viewing Window Parameters** area is used to set the area of the graph the user wishes to see. Use it as follows:

   Specify the desired values for XMin, XMax, YMin, YMax. The applet will do basic error checking. If a non-number is typed in, an error will be displayed, and the focus will remain on the field which caused the error.

   When **Use y-range** is checked, the supplied values for the y-range will be used. Otherwise, the applet will find YMin and YMax from the supplied functions.

   Any change in this area will take effect only after the **Plot** button is pressed.

2. The **Function Information** area is where the functions to be plotted are entered. Use this area as follows:

   To enter a new function, always press the **New Function** button. Then, enter the expression defining the function. The syntax is similar to the syntax used in the Java Math Engine. For example, to defines the sine function, you would type in \( \sin(x) \). Make sure you use \( x \) for the variable in your definition.

   A maximum of 10 functions can be defined at the same time.

   Checking **Active** means the function will show on the graph. Unselecting it means the function will not show.

   Use the + and - buttons to scroll through the list of defined functions.

   Each defined function has a different color assigned to it. The selection is automatic.

   When a function is defined, it is automatically assigned a name of the form \( f_i \) where \( i \) is a number which starts at 1 and is incremented every time a new function is defined. The name a function will be saved under appears to the left of the field where it is defined.

   Once a function is defined, its name can be used in the definition of other functions. For example, if two functions have been defined, the definition of function 3 could be \( f1(x)+x*f2(x) \) Note that we use \( f1(x) \) and not just \( f1 \).

   The functions defined will plot only after **Plot** has been pressed.
**Del Function** will delete the function currently showing. When deleting a function, make sure that it is not used in the definition of another function. An error would occur in this case. For example, if \( f_2(x) = x + f_1(x) \) and \( f_1 \) is deleted, then the definition of \( f_2 \) contains an unknown symbol, \( f_1 \).

3. The **Zoom and Trace** area is where zooming and tracing take place. Use this area as follows:

To zoom in, picture in your mind the rectangular region you would like to zoom in. Left click on one of the corners of this imaginary region. While holding the mouse button down, move it to the opposite corner, then release it. As you move the mouse, a rectangle will be drawn to help you visualize the region. Once the mouse button is released, the graph will redraw, XMin, XMax, YMin, YMax will be updated.

To go back to the original view (XMin = -10, XMax = 10, YMin = -10, YMax = 10), press **Reset Zoom**.

To trace, simply single left click in the graph area. A point will be generated by taking the x-coordinate of the point where you clicked and the y-coordinate on the function currently showing.

To move the point, use the **Right** or **Left** buttons. The point will move on the function currently showing. By selecting a different function, you can select which function you want your point to follow.

The coordinates of the point will be displayed under **X:** and **Y:**.

4. The **Control Buttons** area contains buttons which have a global effect for the applet.

The **Clear All** button erases all the function definitions.

The **Plot** button is used to update the plot area after changes in the other areas have been made.

5. The **Messages** area is where the applet communicates with the user. Error messages as well as user information is displayed there. Errors are displayed in red, while information is displayed in black.

- **Quadratic Regression**

**QUADRATIC REGRESSION APPLET**

This applet has two functions: plotting and finding the quadratic function which best approximates the user supplied data. Entering an appropriate data set, the user plots the data and finds the best fit quadratic function. The function graph is plotted with the data set. The symbolic model and associated error is displayed in the results.

Use it as follows:

1. Put the data to plot and analyze in the Data section. Though the variables default to \( x \) (the independent variable) and \( y \) (the dependent variable), they can be renamed. Renaming them will cause the graph to be updated. The values are entered one per line.

2. The buttons work as follows: **CLEAR** will clear the data, the plot area and the RESULTS area. **PLOT** will plot the data as points. **ANALYZE** will find the quadratic function which best approximates the data, plot it and display its equation in the RESULTS area.

3. The plot area (top, right) will show the plot.

4. The RESULTS area is used to communicate with the user. Errors will be displayed there, for example if the number of values is not the same for both variables. Information will also be displayed there. For example, when the user clicks on **ANALYZE**, the equation of the regression function will appear there, as well as the error associated with this function.

**Instructional Applets**

Instructional applets are designed to demonstrate key concepts used in the Earth Studies modules. They may be used by the student to support the understanding of critical concepts or by the instructor to demonstrate those concepts to the class, a group, or an individual.
• **Defining Trig Functions**

**The Unit Circle and Trigonometric Functions**

The purpose of this applet is to illustrate the relation between the unit circle and how the trigonometric functions are defined. As you drag a point around the unit circle, the applet displays the coordinates and angle direction. Choosing either sine or cosine, the function’s value is displayed as the length of the segment on the unit circle, as well as the corresponding angle.

Use it as follows:

Drag the point around the unit circle, clockwise or counterclockwise, as many times as you want. As you do so, the following should happen:

1. The coordinates of the point will be updated.
2. The point will be in green if it corresponds to a positive angle, otherwise it will be red.
3. Depending on which trigonometric function you have chosen (sine or cosine), its graph will appear and the point on the unit circle will also appear on the graph of the trigonometric function.
4. A red line whose length is the value of the chosen trigonometric function will be displayed on both the unit circle and the graph.
5. If the box “Show Function Values” is checked, the angle and the value of the chosen trigonometric function at that angle will be displayed.
6. As the point on the graph gets ready to leave the graph, the graph will scroll left or right, so that the point remains visible.

• **Derivative and Tangent Line**

The purpose of this applet is to illustrate the relationship of the slope of the tangent to the graph of \( y = f(x) \) at \( x = a \), that can be approximated for a certain value of \( h \) where the secant line becomes the tangent to the graph of \( y = f(x) \) at \( x = a \). This applet illustrates the concept that the slope of the secant approaches the slope of the tangent, allowing the user to enter a function, select the points \((a, f(a))\) and \((a+h, f(a+h))\), to change \( h \) so that the point \((a+h, f(a+h))\) gets closer to the point \((a, f(a))\). Use the applet as follows:

1. If necessary, change the values in the Viewing Window Parameters area, though the default values should be fine in many cases.
2. Enter a function. Nothing can happen unless a function has been entered. Use the same syntax as in the PlotSolve or the Java Math Pad applets.
3. Next, specify the point where the tangent will be drawn.
4. Specify the second point for the secant line.
5. Change the second point defining the secant line. In doing so, the user can verify that as \( h \) approaches \( 0 \), the secant becomes the tangent. This also allows the user to evaluate the expression as \( h \) approaches \( 0 \).

• **Exponential Functions**

**Role of a and b in \( y = ab^x \)**

The purpose of this applet is to illustrate how the values of \( a \) and \( b \) affect the exponential function \( y = a b^x \). Changes made to the general form exponential function, using a scroll bar, will display the affect of the coefficient and base on the resulting graph.

Use it as follows:

1. Use the scroll bar to change the values of \( a \) (between -10 and 10). The exponential function should change as a changes. The value of \( a \) to the left of the scroll bar should also be updated.
2. Use the scroll bar to change the values of b (between 0.1 and 10). The exponential function should change as b changes. The value of b to the left of the scroll bar should also be updated.

• **Function and Derivative**

This applet illustrates the fact that the derivative of a function \( y = f(x) \) at \( x = a \), denoted \( f'(a) \), is the slope of the tangent to the graph of \( y = f(x) \) at \( x = a \). The user can plot a function, and select points on the graph. As the points are selected, the tangent to the graph at the chosen point is drawn, if desired, its slope is displayed and plotted as a new point. If the user selects enough points, the graph of the derivative will become evident. Use the applet as follows:

1. If necessary, change the values in the Viewing Window Parameters area, though the default values should be fine in many cases.

2. Enter a function. Nothing can happen unless a function has been entered. Use the same syntax as in the PlotSolve or the Java Math Pad applets.

3. Next, specify the point where the tangent will be drawn.

4. Specify more points, until the graph of the derivative becomes evident.

• **Graph of Cosine**

**Role of a, b, k and c in \( y = a + b \cos(k(x - c)) \)**

The purpose of this applet is to illustrate how the values of a, b, c and k affect the graph of the cosine function. Changes made in the general form of the Cosine equation will display graph changes of transformation: amplitude, period and phase shift.

Use it as follows:

Use the scroll bar to change the values of a, b, k and c. The graph of the cosine function should change as these values change. The value of each parameter to the left of the scroll bar should also be updated. The changes should happen as follows:

1. Changing a will move the graph up or down by \(|a|\) units.

2. Changing b will change the amplitude (thus the range).

3. Changing k affects the period. The period is \(2 \pi / k\).

4. Changing c will cause the graph to be shifted left or right by \(|c|\) units.

• **Graph of Sine**

**Role of a, b, k and c in \( y = a + b \sin(k(x - c)) \)**

The purpose of this applet is to illustrate how the values of a, b, c and k affect the graph of the sine function. Changes made in the general form of the Sine equation will display graph changes of transformation: amplitude, period and phase shift.

It will function as follows:

Use the scroll bar to change the values of a, b, k and c. The graph of the sine function should change as these values change. The value of each parameter to the left of the scroll bar should also be updated. The changes should happen as follows:

1. Changing a will move the graph up or down by \(|a|\) units.

2. Changing b will change the amplitude (thus the range).

3. Changing k affects the period. The period is \(2 \pi / k\).

4. Changing c will cause the graph to be shifted left or right by \(|c|\) units.
• **Line Slope**

**Understanding the Slope of a Line**

The purpose of this applet is to understand the notion of “the slope of a line”. By selecting two points on the graph, the point coordinates and quotient rise/run is displayed, as well as the triangle symbolizing the relationship. Point positions can be changed and the information is updated.

Use this applet as follows:

1. If you get lost, look in the "MESSAGES" area, you will always be told what to do next. Error messages will also be displayed there.
2. Double-click in the grid area to get a first point. Note that once you have selected a point, its coordinates will appear on the left of the applet window, under POINTS.
3. Double-click a second time to get a second point. Several things will then happen. First, the coordinates of the new point will appear on the left of the applet window, under POINTS. Then, the quotient rise/run and the value of the slope will be displayed on the left of the applet window, under SLOPE. Finally, the line through the two points will be drawn as well as the triangle symbolizing the rise and the run.
4. You can now click on either point and drag it along the line. You will see the point move, the triangle will be redrawn, the coordinates of the point being dragged will be updated. The slope should remain the same.
5. If you either click outside the grid area, or try to drag anything other than one of the point, an error message will be displayed.

• **Linear Functions**

**ROLE of m and b in y = m x + b**

The purpose of this applet is to illustrate how the values of m and b affect the line y = m x + b. Using a scroll bar to change the slope m and/or intercept b, the line position moves to reflect those changes.

Use it as follows:

1. Use the scroll bar to change the values of m (between -10 and 10). The line should change as m changes. The value of m to the left of the scroll bar should also be updated.
2. Use the scroll bar to change the values of b (between -10 and 10). The line should change as b changes. The value of b to the left of the scroll bar should also be updated.

**Quadratic Functions Standard**

**Role of a, b and c in y = a x^2 + b x + c**

The purpose of this applet is to illustrate how the values of a, b and c affect the parabola y = a x^2 + b x + c. Using a scroll bar to change the coefficients and constant, the parabola's position moves to reflect those changes.

Use it as follows:

1. Use the scroll bar to change the values of a (between -10 and 10). The opening of the parabola should change as a changes. The value of a to the left of the scroll bar should also be updated.
2. Use the scroll bar to change the values of b and c (between -10 and 10). The parabola should change as b and c change. The value of b and c to the left of the scroll bar should also be updated.
• **Quadratic Functions Vertex**

**Role of a, h and k in \( y = a (x - h)^2 + k \)**

The purpose of this applet is to illustrate how the values of \( a \), \( h \) and \( k \) affect the parabola \( y = a(x - h)^2 + k \). Using a scroll bar to change the coefficients and constant in the graphing form of a quadratic, the parabola’s position moves to reflect those changes.

Use it as follows:

1. Use the scroll bar to change the values of \( a \) (between -10 and 10). The opening of the parabola should change as \( a \) changes. The value of \( a \) to the left of the scroll bar should also be updated.
2. Use the scroll bar to change the values of \( h \) and \( k \) (between -10 and 10). The position but not the shape of the parabola should change as \( h \) and \( k \) change. The value of \( h \) and \( k \) to the left of the scroll bar should also be updated.

• **Radians and Unit Circle**

**The Unit Circle and Angles**

The purpose of this applet is to illustrate the relation between a point on the unit circle, its coordinates and the angle between the x-axis and the line through the origin and the point. By dragging a point around the unit circle, the coordinates of the point and the angle values will be displayed in either degrees or radians.

Use it as follows:

Drag the point around the unit circle, clockwise or counterclockwise, as many times as you want. As you do so, the following should happen:

1. The coordinates of the point will be updated.
2. The value of the angle between the x-axis and the line through the origin and the point will show only if you have selected to display it. In this case, it will be displayed in the chosen units (degrees or radians).
3. The point will be in green if it corresponds to a positive angle, otherwise it will be red.

**Experimentation Applets**

The experimentation applets are designed to allow the student to find the “best fit” model by trying various approximated coefficients and constants in the standard form of the selected model. The student can adjust the model, viewing their results both symbolically and graphically.

• **Fitting Cosine**

**Fitting Trigonometric (Cosine Function) Data Applet**

This applet has two functions: First, it can be used to plot user supplied data. It can also be used to test if a user supplied trigonometric (cosine) function (a function of the form \( y = a + b \cos(k(x - c)) \)) fits the given data by plotting the function. Plotting an appropriate data set supplied by the user, the cosine function coefficients selected are then entered and tested by graphing with the data set to view the fit of the function.

Use it as follows:

1. The sequence of operations must be as follows:

The user must enter the data to plot in the DATA area. Though the variables default to \( x \) (the independent variable) and \( y \) (the dependent variable), they can be renamed. Renaming them will cause the graph to be updated. The values are entered one per line. Error messages will be displayed in the MESSAGES area if the data does not have the correct format, if the data is not a number.
The user can then plot the data by pressing the "PLOT DATA" button.
The user supplies the coefficients defining the function to test.
The user plots the function by pressing the "TRY" button.

2. The buttons work as follows:
- **CLEAR** will clear the DATA area, the PLOT area and the MESSAGES area.
- **PLOT DATA** will plot the data as points.
- **TRY** will plot the exponential function corresponding to the coefficients supplied by the user.

3. The MESSAGES area is used to communicate with the user. Errors will be displayed there, for example if the number of values is not the same for both variables. Information will also be displayed there.

- **Fitting Exponential Functions**

**FITTING EXPONENTIAL DATA APPLET**

This applet has two functions: First, it can be used to plot user supplied data. It can also be used to test if a user supplied exponential function (a function of the form \( y = a \cdot b^x \) ) fits the given data by plotting the function. Plotting an appropriate data set supplied by the user, the exponential function coefficients selected are then entered and tested by graphing with the data set to view the fit of the function.

Use it as follows:

1. The sequence of operations must be as follows:

   The user must enter the data to plot in the DATA area. Though the variables default to x (the independent variable) and y (the dependent variable), they can be renamed. Renaming them will cause the graph to be updated. The values are entered one per line. Error messages will be displayed in the MESSAGES area if the data does not have the correct format, if the data is not a number.

   The user can then plot the data by pressing the "PLOT DATA" button.

   The user supplies the coefficients defining the function to test.

   The user plots the function by pressing the "TRY" button.

2. The buttons work as follows:
- **CLEAR** will clear the DATA area, the PLOT area and the MESSAGES area.
- **PLOT DATA** will plot the data as points.
- **TRY** will plot the exponential function corresponding to the coefficients supplied by the user.

3. The MESSAGES area is used to communicate with the user. Errors will be displayed there, for example if the number of values is not the same for both variables. Information will also be displayed there.

4. The MESSAGES area is used to communicate with the user. Errors will be displayed there, for example if the number of values is not the same for both variables. Information will also be displayed there.

- **Fitting Linear Functions**

**FITTING LINEAR DATA APPLET**

This applet has two functions: First, it can be used to plot user supplied data. It can also be used to test if a user supplied linear function (a function of the form \( y = m \cdot x + b \) ) fits the given data by plotting the function. Plotting an appropriate data set supplied by the user, the linear function coefficient and constant selected are then entered and tested by graphing with the data set to view the fit of the function.
Use it as follows:

The sequence of operations must be as follows:

1. The user must enter the data to plot in the DATA area. Though the variables default to \( x \) (the independent variable) and \( y \) (the dependent variable), they can be renamed. Renaming them will cause the graph to be updated. The values are entered one per line. Error messages will be displayed in the MESSAGES area if the data does not have the correct format, that is if the data is not a number.

The user can then plot the data by pressing the "PLOT DATA" button.

The user supplies the coefficients defining the function to test.

The user plots the function by pressing the "TRY" button.

2. The buttons work as follows:
   - CLEAR will clear the DATA area, the PLOT area and the MESSAGES area.
   - PLOT DATA will plot the data as points.
   - TRY will plot the linear function corresponding to the coefficients supplied by the user.

3. The PLOT area (top, right) will show the plot.

The MESSAGES area is used to communicate with the user. Errors will be displayed there, for example if the number of values is not the same for both variables. Information will also be displayed there.

- **Fitting Quadratic Functions**

**FITTING QUADRATIC DATA APPLET**

This applet has two functions: First, it can be used to plot user supplied data. It can also be used to test if a user supplied quadratic function (a function of the form \( y = a x^2 + b x + c \)) fits the given data by plotting the function. Plotting an appropriate data set supplied by the user, the quadratic function coefficients and constant selected are then entered and tested by graphing with the data set to view the fit of the function.

Use it as follows:

The sequence of operations must be as follows:

1. The user must enter the data to plot in the DATA area. Though the variables default to \( x \) (the independent variable) and \( y \) (the dependent variable), they can be renamed. Renaming them will cause the graph to be updated. The values are entered one per line. Error messages will be displayed in the MESSAGES area if the data does not have the correct format, that is if the data is not a number.

The user can then plot the data by pressing the "PLOT DATA" button.

The user supplies the coefficients defining the function to test.

The user plots the function by pressing the "TRY" button.

2. The buttons work as follows:
   - CLEAR will clear the DATA area, the PLOT area and the MESSAGES area.
   - PLOT DATA will plot the data as points.
   - TRY will plot the quadratic function corresponding to the coefficients supplied by the user.

3. The PLOT area (top, right) will show the plot.

4. The MESSAGES area is used to communicate with the user. Errors will be displayed there, for example if the number of values is not the same for both variables. Information will also be displayed there.
Fitting Sine Functions

FITTING TRIGONOMETRIC (SINE FUNCTION) DATA APPLET

This applet has two functions: First, it can be used to plot user supplied data. It can also be used to test if a user supplied trigonometric (sine) function (a function of the form \( y = a + b \sin(k(x - c)) \)) fits the given data by plotting the function. Plotting an appropriate data set supplied by the user, the sine function coefficients selected are then entered and tested by graphing with the data set to view the fit of the function.

Use it as follows:

The sequence of operations must be as follows:

1. The user must enter the data to plot in the DATA area. Though the variables default to \( x \) (the independent variable) and \( y \) (the dependent variable), they can be renamed. Renaming them will cause the graph to be updated. The values are entered one per line. Error messages will be displayed in the MESSAGES area if the data does not have the correct format, that is if the data is not a number.

The user can then plot the data by pressing the "PLOT DATA" button.

The user supplies the coefficients defining the function to test.

The user plots the function by pressing the "TRY" button.

2. The buttons work as follows:

   CLEAR will clear the DATA area, the PLOT area and the MESSAGES area.

   PLOT DATA will plot the data as points.

   TRY will plot the trigonometric function corresponding to the coefficients supplied by the user.

3. The PLOT area (top, right) will show the plot.

4. The MESSAGES area is used to communicate with the user. Errors will be displayed there, for example if the number of values is not the same for both variables. Information will also be displayed there.
Chapter 4  Inquiry with Earth Studies

Inquiry-Based Instruction

This section of the guidebook gives an overview of information regarding inquiry, good questions, assessment, and web sites supporting an inquiry approach.

FAQ  How do I use an inquiry approach?

Inquiry in the Math Classroom Using the Earth Math Studies

- Introduction to Inquiry-based Instruction  4-1
- Good Questions  4-4
- Assessment and Adaptation  4-5
- Web Sites and Bibliography  4-7

Instructors using Earth Math modules will find a description of inquiry-based instruction and good questions. A discussion of the components inherent in the approach, assessment and adaptation of the modules are included. Finally, additional web sites with information for teachers of the inquiry approach and a Bibliography used for this chapter complete the information in this last chapter.
Chapter 4 Inquiry-Based Instruction

Using Inquiry-Based Learning

Introduction

A philosophical and key feature of the National Standards movement in both Mathematics and Science Education is the focus on learner-centered and inquiry-based processes for pedagogical enhancement. Mathematics Standards of the NCTM, AMATYC, MAA and others emphasize the necessity for students to gain experience with inquiry in a data rich environment. For example, the NCTM Principles and Standards for School Mathematics, Chapter 3 states

“Instructional programs … should enable all students to—

- formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- select and use appropriate statistical methods to analyze data;
- develop and evaluate inferences and predictions that are based on data; …”

(Principles and Standards for School Mathematics, Chapter 3; http://standards.nctm.org/document/chapter3/data.htm)

One of the Science Standards features call for students to engage in inquiry “to develop:

- understanding of scientific concepts,
- appreciation of ‘how we know’ what we know in science,
- understanding of the nature of science,
- skills necessary to become independent inquirers about the natural world, and
- dispositions to use the skills, abilities, and attitudes associated with science.”

(National Science Education Standards; 1996; p.105)

Guidance in the meaning and importance of inquiry in mathematics and science as a multi-faceted approach, creating pedagogical and substantive examples showing what it looks like in practice, and providing classroom ready resources are necessary to meet the Standards aiding educators in understanding and implementing sound principles of inquiry. The Earth Math Projects fulfill these goals in both the approach and pedagogical resources created for educators and students alike.

A Definition of Inquiry

What is Inquiry-Based Learning?

Inquiry as a learner-centered process is central to its definition – what students know and want to learn are the foundation for learning. Several definitions of inquiry point out this natural building of ideas:

- "For students, this method of learning ends the listen-to-learn paradigm of the classroom and gives them a real and authentic goal challenges to overcome. For the teacher, inquiry-based education ends their paradigm of talking to teach and recasts them in the role of a colleague and mentor engaged in the same quest as the other younger learners around."
  (http://www.ncsa.uiuc.edu/Cyberia/DVE/FusionDVE/html/inquiry_based_education.html)

- "Inquiry is an approach to learning that involves a process of exploring the natural or material world, that leads to asking questions and making discoveries in the search for new understandings."
  (http://www.exploratorium.edu/IFI/resources/inquirydesc.html)

- Inquiry education is where structure meets fluidity, where we can create opportunities for students to be engaged in active learning based on their own questions. (From a small group discussion on "Inquiry in Action" at The Inquiry Page Workshop on Feb 21, 2001.)
Inquiry-based learning is a dynamic process, encompassing a whole range of models, including some important aspects that should be supported in a learning environment. Inquiry often leads to the creation of new ideas and constructively communicating those ideas is central to the whole process. The process should include the following concepts:

**ASK**
Meaningful questions inspired by genuine curiosity about real-world experiences. A problem or question provides the focus and the learner begins to define or describe what it is. Questions are redefined throughout the learning process.

**INVESTIGATE**
Putting the curious impulse into action and beginning to gather information: researching resources, studying, crafting an experiment, observing, or interviewing; recasting the question, refining a line of query, or following a new lead – a self-motivated process that is wholly owned by the engaged learner.

**CREATE**
The learner makes connections and begins to synthesize meaning to form new knowledge. The learner shapes new thoughts, ideas, and theories outside of previous experience.

**DISCUSS**
Learners share their new ideas with others in a community-building process of shared knowledge, where the meaning of an individual’s investigation takes on greater relevance in the context of the learner’s society.

**REFLECT**
Taking time to look back at the question, the research path and the conclusions made, the learner takes inventory, makes observations, and makes new decisions. New questions come to light and so the questions begin again.

(Adapted from Winters, Tina, & Hollweg, Karen. 2001. Inquiry in the Standards-Based Science Classroom. ENC Focus 8(2) p.20-22)

The Earth Studies modules support this process of inquiry-based learning. The modules are designed to facilitate the formulation of good questioning and mathematical modeling skill, within the scientific inquiry of the topics presented.

Each Study will have three components:
- the **text**, which will provide information about a particular issue and ask questions;
- the **Menu** will be in two parts: the **Tool Chest**, which will contain the applets to perform the mathematical operations; and a **Math Topics** list, which will contain links to explanations of relevant mathematical topics;
- the **journal**, which will be the place where you can record your thoughts, answers, and mathematical solutions.

It is in the text of each module that the student will find questions and topic information to begin this process, the ASK and INVESTIGATE steps are found in the beginning of the module.

The text of the Earth Studies module will be presented in four sections: **Comprehension; Acquisition; Application; and Reflection**.

- The **Comprehension** section will provide information about the issue and ask pertinent questions for you to think about and express your own ideas regarding the issue. This section corresponds to the first step in the inquiry process, that of asking relevant questions about real-world experiences. The focus presented in this section begins with the student’s understanding of the topic from their own experience. The learner will begin to consider the issue and consequences of some preliminary predictions. Questions regarding the student’s understanding of the world and global perspective are used in this section, for example in the Population module, the following questions are included.

**Questions**
A. Do you think that the population of the World is increasing or decreasing?
B. What have you noticed recently that led you to your answer to A?
C. How do you think population change in the world might influence your life?
D. How do you think mathematics can be used to study population change?
E. What are some reasons for studying population change?
These questions provide a focus for the student’s thoughts, using their experience and thinking about population, as well as what change might be occurring in the population. The questions are designed to develop thought regarding the use of mathematics in the study as well.

- The Acquisition section will provide more detailed information, relevant data, or direct you to a website to acquire more information or data. It is in this section the student will begin to investigate the information provided in the form of real data and experiment with the mathematical tools in the tool chest. Students will also review the mathematical topics in their repertoire and find additional information to add to their understanding of the topic. They may follow this information to new questions or it may lead them to investigate related but separate questions. The self-motivating process is owned by the learner, leading to increased attention to the tools used to answer questions of interest to the engaged learner.

- The Application section will ask specific questions which will require mathematical analysis that will lead to a mathematical model. The model should be able to provide answers to questions raised while investigating the issue and obtaining information. It is in this section that the applets will be used. The students create a model from the given data and find the answers to additional questions in this section. The refinement of the study and prediction from models lead to further questions, where the learner makes connections and begins to synthesize meaning to form new knowledge about the topic. The learner forms new thoughts, ideas, and theories about the topic in a way not previously experienced.

- It is important at this junction that students be provided the opportunity to share these ideas and theories. This is the point in time for the students to report the findings of their investigations in both their journals and in the classroom community. This shared knowledge will contribute to the student's confidence in their thoughts and the individual investigation takes on greater relevance in the context of their learning community. Often ideas are confirmed or the need for further study identified through this community sharing within the classroom. Students can share ideas within the whole class or small groups, in written or oral presentation formats, and include a discussion of the findings.

- The Reflection section will ask questions regarding the reasonableness or validity of your model. The questions in this section ask the student to predict and make conclusions or inferences from the study they have undertaken. For example, the reflection questions from the Calculus module on world grain supply are:

  **Reflection:** Assessing the method, solution and implications
  Do you think that the functions used are appropriate? If not, what might be better?
  Do you think that the solution is reasonable? Why?
  Do you think that the supply function is accurate? Do you think that it is reasonable to assume that this trend will continue? If not, what do you think might happen?
  Would it be appropriate to construct an alternative model?

- The observations and decisions lead to new questions and so the process can begin again. Encouraging students to delve into further study of new questions bring them full circle in the educational process of inquiry.

**Planning and Strategy**

The Earth Studies allow easy planning for inquiry-based and student directed learning. Questions for planning inquiry in the classroom should include the following ideas:

- **Concepts**: What are the important concepts or big ideas in the desired study or topic? How are the concepts defined or categorized in examples?

- **Processes**: What are the processes needed to collect information, analyze data, report and produce findings, and how will you know? Which processes will you include (collecting data, analyzing data, drawing conclusions, representing knowledge)?
Generalizations
Ask how the topic concepts might be linked. What are the relevant relationships, characteristics, similarities or differences, consequences, conditions or contexts, and assumptions that might affect results?

Theory
Ask how the relevant generalizations might be formed to create a theory that is true and important to know.

Products
What product needs to be designed to demonstrate the concept or support the ideas presented? What skills and knowledge must be demonstrated and how does the product provide expected evidence? What range of possibilities and documentation would be necessary?

Assessment
How will you know what the students know? What types of authentic and alternative assessments will you allow? What criteria will you use and what work will be included in the student’s assessment?

Schedule
How will you use your classroom time effectively and optimize available time? How will you organize the classroom or lab? Will you group the students and how?

Plans
What resources will you use and when will you use them? What questions will you ask and how are the resources related to the student’s interests and needs?

The module questions will form the basis for the student inquiry but additional questioning by the instructor will help the students formulate their ideas and theories. Good questions by the instructor can include the following question types.

**Good Questions**
A key task for the instructor of the Inquiry Based materials is the formation of good, meaningful questions, to assist the student in formulating ideas and creating interest or curiosity in the student. The Earth Math materials will begin each module with a set of questions for students to discuss and think about, pertaining to the topic of the module. Questions in the modules are often enough to begin a study, but students and instructors often need to learn more about the way to ask good questions in the inquiry format.

**Question Types**

**Clarifying**
Focus on a specific idea and provide a clear setting for the answer. These require the student to make explicit ideas and meanings.

**Contextual**
Require the students situate their ideas, determine the conditions for the existence of the ideas, and factors that effect meaning.

**Divergent**
Suggest a wide range of responses and can be answered in a variety of ways.

**Extending**
Assist students to extend their understanding of concepts, draw conclusions, and provide reasoning to support those concepts.

**Focusing**
Identify the “big ideas” with concrete examples, definitions, characteristics, and categorize the information.

**Open-Ended**
Require more than one answer or cannot be answered with a “yes” or “no”.

**Organizing**
Determine the organization to the events, compare and contrast, find cause and effect, and follow with a “why” question.

**Shared**
Encourage the student to enter a discussion or conversation with a variety of unique ideas.

**Thought-Provoking**
Require a reasoning, engagement, and insight so these questions provide more than a simple answer.
As an instructor of inquiry-based learning, always request the student to generalize their results and draw a conclusion. Ask students for the reasoning used for the conclusion – support (evidence) for the "why" of the idea or comparisons made. Assist the student in finding different relationships, labeling and identifying attributes, and focus on generalizations. This will facilitate the formulation of the generalization, conclusion, and prediction of new connections for the student.

**Environment**

The process of learning, not just what is learned is important in the use of the Earth Math modules. The focus of inquiry-based learning is on the process, not the product of learning. The environment you provide should support the questions and paths the students choose, while you facilitate the process. Students taking responsibility, making decisions, initiating the inquiry and learning while you as instructor facilitate the collaboration make inquiry-based learning a dynamic environment.

The characteristics of the environment should include the feeling of success, support, appreciation, recognition, and visible signs of learning. The affective values of acceptance of difference and critical awareness of political consciousness, understanding, relationships, and societal goals should be established throughout your classroom.

As the student develops the concepts, use your questions to guide the focus and direction of inquiry. Include relationships and extend the thoughts beyond simple answers as you complete analyses. Do not repeat the student's responses, as this could encourage others not to listen. Always as open-ended questions and listen carefully to all responses.

For focusing questions, address only one situation at a time – simplify complex situations into several distinct steps to the final conclusion. When a response is too general, ask for an example to clarify the student's understanding and conclusion. Always ask for reasoning, explanation that is descriptive in support of the response.

**Interpretation of Data**

The interpretation of data will require students to first recall information from past or recent experience, gleaned from the reading, demonstrations, and assignments. Students need to determine what is relevant in responding to the focusing questions you provide. Ask for a variety of information, be accepting of all responses, and ask for clarification with specific information where necessary.

Next ask students to state causes or effects of selected datum to identify evidence or support for their inference, and establish links or relationships. Never establish those links yourself but allow the discussion and findings, as well as your facilitating questions to lead the student to an appropriate, yet open-ended conclusion. Encourage the students to use data to establish inferences and generalizations about the issue, testing their conceptual understanding of the relationships. Students can then establish an understanding of the complexity and interrelatedness of the events, examining the long-term implications. The students then draw a conclusion based on the evidence presented.

The students should then be encouraged to apply the generalization to a prediction of future events, using their earlier work. Predictions infer the effects of a given situation, using facts or generalizations to form the basis of that prediction. An objective of this prediction is for the student to apply previously learned information to a new situation or to solve a problem. Students should provide factual and logical support for their prediction, identifying conditions that make the prediction plausible. The final step in the interpretation of data is to extend the prediction to consequences that extend from the original situation.

**Assessment of Learning**

No chapter on inquiry-based learning would be complete without some discussion about the assessment of learning in this environment. Assessment can be done for the individual student or student groups, or a combination of both. The easiest assessments are done by determining what questions the students must answer and in what format, completed in the journal or a portfolio.
Designing a set of points or rubric for evaluating each part should provide assessment of evidence for conceptual change, growth, reflection, decision-making, and understanding.

Advise students to reflect on what they have come to know, the processes and concepts developed, and what best reflects the processes and solutions obtained. Questions you might like students to answer are contained within the modules, but you might also include documentation of the students’ problem-solving, individual research or questions, and models obtained during their work.

Students can make comparisons of what they like the most, how items are similar or different and account for those similarities or differences, what they need to know further and how they would learn it, and document changes in processes they undertake. You can also ask them to note the changes in their thinking that has occurred, what influenced the change, and what they have come to know about their “self”. They can order the series of questions they raised and the answers determined, forming a sequence of steps to the solution of the problem.

Each of the required questions and criteria for assessment should be clearly documented for the student as they begin the module. If you wish a complete, step-by-step sequence of answers and solutions, this should be clearly stated with examples, and the rubric for evaluation or points allotted each portion of the study should be provided the student. If the assessment will take the form of a journal or portfolio, clearly document the required components and criteria for assessing each part. In my experience, the more clearly you document the assessment criteria and expectations to the student, the better the results you obtain from the students.

You can make the documentation from the student by individual or by group. I recommend each student have their own assessment, although a group score might be part of that, either from other group members, or from you as an assessment of how they have worked with their group members (their participation). All assessment should take the form of improving learning, so you might also want to include a portion for the student to assess themselves, or write items for improvement prior to the next module or study undertaken. Many students begin without knowing how to ask good questions or find related information. The modules assist the students in doing this, so provision should be made for including improvement in their assessment, either by you or the student themselves.

**Adapting Modules to Your Students**

All of the Earth Studies modules can be adapted to your course and learning environment. The modules can be given as individual or group projects in a laboratory component, or integrated into the regular classroom time as an application of previous learning. Modules are designed to be done independently, so a student can self-select one of interest, or you can design a series of labs for your students to work with that follow your standard curriculum.

One of the most interesting ways to use the Earth Math Studies are to find different but related data from your local area (or have your students find this data) and assign the students to conduct a similar inquiry with the local or regional data. For example, the population data used with World and United States population data can be brought into more relevant focus for students when they use the population data of their city, county, or state. They can do a comparison of the rate of population increase (or decrease) and discuss this in relationship to their own experience.

Since the modules are written with environmental issues, many of the studies can be easily revised to use data for the issues of the students’ home region. You will find it easy to replace the information and questions to those of your region, as the questions are open-ended and merely focus on student thinking. Use the Earth Studies standard format as a guide and replace each part with the questions, data and information, and reflection for a local issue of your selection. Students will often find their own data as well as questions as they work on the web site modules, so encourage extended exploration when this occurs.

**Web Sites for Inquiry-Based Learning**

The following web sites have both information for further study and pedagogical resources for the teacher of an inquiry-based curriculum component.
Web Sites

Using the Internet to Promote Inquiry-based Learning
This link provides guidance in using the internet and information for inquiry-based learning.
http://www.biopoint.com/msla/links.html

Beyond School Walls
Read about these inquiry and problem solving activities that take place outside the classroom in the field.
http://www.enc.org/topics/inquiry/beyond/

Classroom Activities
Read about activities that teachers have implemented in their classrooms to promote inquiry and problem solving.
http://www.enc.org/topics/inquiry/activities/

Inquiry Page
This page provides a wealth of different topics related to inquiry learning and activities.
http://www.inquiry.uiuc.edu/

Inquiry and the National Science Education Standards: A Guide for Teaching and Learning will provide you with a rich resource of personal examples and guidelines to aid you in designing and carrying out inquiry-based science activities in your classroom. You may read the report free of charge online (www.nap.edu/books/0309064767/html/).

The second addendum to the Standards, titled Classroom Assessment and the National Science Education Standards, will be available in 2001. For more information about this and other resources from the National Research Council, contact Tina Winters at T_Winters@nas.edu

Disney Learning Partnership; Concept to Classroom Resources; Inquiry-based Learning; Books, Articles, and Websites
http://www.thirteen.org/edonline/concept2class/w6-resources.html

Bibliography

Principles and Standards for School Mathematics, Chapter 3;

National Science Education Standards; 1996; p.105

Inquiry in the Standards-Based Science Classroom: A New Resource for Teachers and Teacher Educators; A new guide helps teachers provide standards-based science to students of all ages. The full text is available free online. Tina Winters and Karen Hollweg,

National Research Council

References


The Research Base Supporting Inquiry Learning

The following list of resources is a sampling of the research available that supports inquiry learning. Many more references are cited in the full report.


**Answers to Studies**

The Earth Math Projects has links for the instructor to access the answers to the Earth Math Projects. The Introductory Module has the answers embedded in the study to demonstrate the integration of the materials. It is the only module with the answers embedded, the rest of the modules are linked to the answers through the web site. Instructors will have the answers to the study questions and appropriate models as produced through the use of the study in its online form, but these are not available to students online.